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Abstract

This article intends to show that Maxwell equation can be derived from the basic Electromagnetic equations and that these ones are just result of applying vector operations to the basic formulas. The first equation could also contain wrong sign.

This all is showed on exactly and clear way by introduction of basic vector integral transformations only.

DERIVATION OF MAXWELL EQUATIONS AND THEIR CORRECTIONS

We take for granted that Maxwell¹ equations are completely correct, but is it true? In the further text we will check Maxwell equations by reversal derivation from the Faraday² law of electromagnetic induction, Biot³-Savart⁴ law and Coulomb⁵ law.

Faraday's law is given by following equation:

$$U = - \frac{\partial \Phi}{\partial t} \quad (1)$$

Whereas:

U = electrical potential,
Φ = magnetic flux,
t = time.

Formula of Biot-Savart law is:

$$d\vec{B} = \frac{\mu}{4 \cdot \pi} \cdot \frac{I \cdot d\vec{\ell} \times \hat{r}}{r^2} \quad (2)$$

Whereas:

$\vec{B}$ = magnetic field,

¹ James Clerk Maxwell, 1831-1879

² Michael Faraday, 1791-1867

³ Jean-Baptiste Biot, 1774-1862

⁴ Felix Savart, 1791-1841

⁵ Charles Coulomb, 1736-1806

- μ = magnetic permeability,
- I = electrical current,
- ℓ = length,
- $\vec{r}$ = radius vector,
- $\hat{r}$ = normalized radius vector, i.e. $\frac{\vec{r}}{|\vec{r}|}$.

Coulomb law is:

$$\vec{F}_{12} = -\frac{1}{4 \cdot \pi \cdot \varepsilon} \cdot \frac{Q_1 \cdot Q_2}{\vec{r}^2} \cdot \hat{r} \quad (3)$$

Whereas:

- $\vec{F}_{12}$ = force between two charged bodies,
- Q_1 = first charge,
- Q_2 = second charge,
- ε = electrostatic permeability,
- $\vec{r}$ = distance vector,
- $\hat{r}$ = normalized distance vector – unit vector, i.e. $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$.

Regarding formula (3) we can derived following formula for electric field from the definition of field intensity:

$$\vec{E}_1 = \frac{\vec{F}_{12}}{Q_2} = -\frac{1}{4 \cdot \pi \cdot \varepsilon} \cdot \frac{Q_1}{\vec{r}^2} \cdot \hat{r} \quad (4)$$

Whereas:

- $\vec{F}_{12}$ = force between two charged bodies,
- $\vec{E}_1$ = electric field of the first body,
- Q_1 = first charge given in coulombs,
- ε = electrostatic permeability of medium,
- $\vec{r}$ = distance vector,
- $\hat{r}$ = normalized distance vector – unit vector, i.e. $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$.

Finally:

$$\vec{E} = -\frac{1}{4 \cdot \pi \cdot \varepsilon} \cdot \frac{Q}{\vec{r}^2} \cdot \hat{r} \quad (5)$$

Connection between voltage potential U and electric field $\vec{E}$ is given by the following equation:

$$\vec{E} = -\vec{\nabla}U \quad (6)$$

I.e.

$$U_{a,b} = U_b - U_a = \int_a^b \vec{E} \cdot d\vec{\ell} \quad (7)$$

Definition formula of magnetic flux is:

$$\Phi = \iint_S \vec{B} \cdot d\vec{S} \quad (8)$$

Regarding (1), (6) and (8) we can compute circular integral over the contour:

$$\oint_{\ell} \vec{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S} \quad (9)$$

The following transformation is always possible:

$$\oint_{\ell} \vec{E} \cdot d\vec{\ell} = \iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} \quad (10)$$

Regarding (9) and (10) is obtained:

$$\iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S} \quad (11)$$

If we assume that there is no relativistic transformation of surface perpendicular to velocity, we are able to directly obtain first Maxwell equation:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (12)$$

It means that Faraday's equation is completely identical to the incomplete form of first Maxwell's equation.

We can do that with an additional operator's relation that is always valid in E^3 space:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \quad (13)$$

I.e.

$$\frac{\partial}{\partial t} = \frac{d}{dt} - (\vec{v} \cdot \vec{\nabla}) \quad (14)$$

Whereas $\vec{v}$ is velocity vector. Regarding (14) equation (12) becomes:

$$\vec{\nabla} \times \vec{E} = (\vec{v} \cdot \vec{\nabla}) \cdot \vec{B} - \frac{d\vec{B}}{dt} \quad (15)$$

There is also a general vector identity:

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = \vec{A} \cdot \vec{\nabla} \vec{B} - \vec{B} \cdot \vec{\nabla} \vec{A} + (\vec{B} \cdot \vec{\nabla}) \cdot \vec{A} - (\vec{A} \cdot \vec{\nabla}) \cdot \vec{B} \quad (16)$$

The above equation could be transformed into more suitable form:

$$(\vec{B} \cdot \vec{\nabla}) \cdot \vec{A} = -\vec{A} \cdot \vec{\nabla} \vec{B} + \vec{B} \cdot \vec{\nabla} \vec{A} + (\vec{A} \cdot \vec{\nabla}) \cdot \vec{B} + \vec{\nabla} \times (\vec{A} \times \vec{B}) \quad (17)$$

Regarding equation (17), the equation (15) is be written in the following form:

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{B} \times \vec{v}) + (\vec{B} \cdot \vec{\nabla}) \cdot \vec{v} + (\vec{\nabla} \cdot \vec{B}) \cdot \vec{v} - \vec{B} \cdot (\vec{\nabla} \cdot \vec{v}) - \frac{d\vec{B}}{dt} \quad (18)$$

When $\vec{B}$ field is homogenous and $\vec{v}$ is constant in time, the equation (18) becomes:

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{B} \times \vec{v}) \quad (19)$$

$\Rightarrow$

$$\vec{E} = \vec{B} \times \vec{v} = -\vec{v} \times \vec{B} \quad (20)$$

This is well known formula of macroscopic effect and it shows that signs in equation (12) may not be fully correct. It appoints on possibility that equation (12) has wrong sign.

We cannot derive the second Maxwell equation (21) from (2) so easy as we derived (12) from (1) because formula (1) seems to be only a good approximation of second Maxwell equation. Thus we have to make a temporary assumption that following second Maxwell equation is correct:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \cdot \frac{\partial \vec{E}}{\partial t} \quad (21)$$

Definition of electrical current is:

$$I = \frac{dQ}{dt} \quad (22)$$

For now, let us take for granted that is:

$$c = \frac{1}{\sqrt{\mu \cdot \varepsilon}} \quad (23)$$

Equation (21) by involving (14) becomes:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \cdot \left(\frac{d}{dt} \vec{E} - (\vec{v} \cdot \vec{\nabla}) \cdot \vec{E} \right) \quad (24)$$

By involving of (17) into (24) we obtain:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \cdot \left(\frac{d}{dt} \vec{E} - \vec{v} \cdot \vec{\nabla} \vec{E} + \vec{E} \cdot \vec{\nabla} \vec{v} - (\vec{E} \cdot \vec{\nabla}) \cdot \vec{v} \right) - \frac{1}{c^2} \cdot (\vec{\nabla} \times (\vec{E} \times \vec{v})) \quad (25)$$

If the $\vec{E}$ is homogenous and $\vec{v}$ is constant in time we have:

$$\vec{v} \times \vec{B} \approx -\frac{1}{c^2} \cdot (\vec{v} \times (\vec{E} \times \vec{v})) \quad (26)$$

$\Rightarrow$

$$\vec{B} \approx -\frac{1}{c^2} \cdot (\vec{E} \times \vec{v}) = \frac{\vec{v} \times \vec{E}}{c^2} \quad (27)$$

After differentiation on t and regarding (5) and (23) we obtain:

$$d\vec{B} \approx \mu \cdot \varepsilon \cdot \vec{v} \times d\vec{E} = \mu \cdot \varepsilon \cdot \frac{d\vec{\ell}}{dt} \times \frac{1}{4 \cdot \pi \cdot \varepsilon} \cdot \frac{dQ}{\vec{r}^2} \cdot \hat{r} \quad (28)$$

Regarding (22) we have:

$$d\vec{B} \approx \frac{\mu}{4 \cdot \pi} \cdot \frac{I \cdot d\vec{\ell} \times \hat{r}}{\vec{r}^2} \quad (29)$$

Formula (2) and (29) are identical. It shows that equation (2) is approximation of equation (21).

In regards with formula (5), (22) and (23), formula (2) can be rearranged in the following way:

$$d\vec{B} = \frac{\mu}{4 \cdot \pi} \cdot \frac{I \cdot d\vec{\ell} \times \hat{r}}{\vec{r}^2} = \frac{\mu}{4 \cdot \pi} \cdot \frac{dQ}{dt} \cdot \frac{d\vec{\ell} \times \hat{r}}{\vec{r}^2} = \frac{\varepsilon}{\varepsilon} \cdot \frac{\mu}{4 \cdot \pi} \cdot \frac{dQ}{\vec{r}^2} \cdot \frac{d\vec{\ell}}{dt} \times \hat{r} \quad (30)$$

After integration we obtain:

$$\vec{B} = \frac{1}{c^2} \cdot \vec{v} \times \vec{E} \quad (31)$$

This is a formula of a macroscopic effect that shows us that formulas (2) and (21) have all signs correct.

Formulas (20) and (31) are interesting because they act from right to left, i.e. on the right side is real field and on the left one is effective one noticed by the observer.

Let we imagine a rotating ring with a charge in its center. After formula (31) is involved into formula (20), it is obtained:

$$\vec{E}_{ind} = \vec{E}_{\parallel} + \vec{E}_{\perp} = \frac{(\vec{v} \times \vec{E}) \times \vec{v}}{c^2} = \frac{\vec{v}^2 \cdot \vec{E}}{c^2} - \frac{(\vec{v} \cdot \vec{E}) \cdot \vec{v}}{c^2} \quad (32)$$

Induced electric field is consisted of two components: one parallel with velocity and other one parallel to field.

Regarding all seen in the text above we can conclude first Maxwell equation (12) and the second one (21) yield the first iteration of the manifestation. If we involve equation (21) into (12) we obtain:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E}_{ind} = -\frac{1}{c^2} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} \quad (33)$$

I.e.:

$$\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}_{ind}) - \Delta \cdot \vec{E}_{ind} = -\frac{1}{c^2} \cdot \frac{\partial^2 \vec{E}^2}{\partial t^2} \quad (34)$$

If there is no electric current in vacuum we can conclude that the first part of above equation is equal to zero and thus we have:

$$\Delta \cdot \vec{E}_{ind} = \frac{1}{c^2} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} \quad (35)$$

Above equation could be recursively extended:

$$\Delta \cdot \vec{E}_{k+1} = \frac{1}{c^2} \cdot \frac{\partial^2 \vec{E}_k}{\partial t^2} \quad (36)$$

I.e.:

$$\vec{E}_{k+1} = \frac{1}{c^2} \cdot \Delta^{-1} \cdot \frac{\partial^2 \vec{E}_k}{\partial t^2} \quad (37)$$

The total magnitude of electric field can be computed from the following equation now:

$$\vec{E}_{tot} = \sum_{k=0}^{\infty} \vec{E}_k \quad (38)$$

More precisely, regarding (37) we have:

$$\vec{E}_{tot} = \sum_{k=0}^{\infty} \left(\Delta^{-1} \cdot \left(\frac{1}{c} \cdot \frac{\partial}{\partial t} \right)^2 \right)^k \cdot \vec{E}_0 \quad (39)$$

Solution of equation (39) is:

$$\vec{E}_{tot} = \frac{1}{1 - \Delta^{-1} \cdot \left(\frac{1}{c} \cdot \frac{\partial}{\partial t} \right)^2} \cdot \vec{E}_0 \quad (40)$$

If there is no acceleration in homogenous field, equation (14) becomes:

$$\frac{\partial^2}{\partial t^2} = \vec{v}^2 \cdot \Delta \quad (41)$$

Regarding equation (41) equation (40) becomes:

$$\vec{E}_{\text{tot}} = \frac{1}{1 - \frac{\vec{v}^2}{c^2}} \cdot \vec{E}_0 \quad (42)$$

If the equation (40) is applied on (12), it is obtained:

$$\vec{E} = \frac{1}{1 - \Delta^{-1} \cdot \left(\frac{1}{c} \cdot \frac{\partial}{\partial t} \right)^2} \cdot \text{Vecpot} \left(-\frac{\partial \vec{B}}{\partial t} \right) \quad (43)$$

It is more accurate version of Maxwell equation (12) for macroscopic field's manifestation. On the similar way can be found analog correction to equation (21).

Let us start from the equation (5). After integration over the sphere containing the charge we:

$$\oint_S \vec{E} \cdot d\vec{S} = -\frac{1}{4 \cdot \pi \cdot \epsilon} \cdot \oint_S \frac{Q}{r^2} \cdot \hat{r} \cdot d\vec{S} \quad (44)$$

$\Rightarrow$

$$\iiint_V (\vec{\nabla} \cdot \vec{E}) \cdot dV = -\frac{1}{4 \cdot \pi \cdot \epsilon} \cdot \frac{Q}{r^2} \cdot 4 \cdot \pi \cdot r^2 \quad (45)$$

$\Rightarrow$

$$\iiint_V (\vec{\nabla} \cdot \vec{E}) \cdot dV = -\frac{Q}{\epsilon} \quad (46)$$

$\Rightarrow$

$$\vec{\nabla} \cdot \vec{E} = -\frac{\rho_Q}{\epsilon} \quad (47)$$

Whereas:

- $\vec{E}$ = Electric field,
- ρ_Q = Density of electric charge,
- ϵ = Electric permeability of medium.

TEST

Let we imagine a charge in the middle of a ring, the force that act to the charge regarding formulas (5) and (32) should be a reactive force:

$$\vec{F}_{\text{ind}} = Q \cdot \vec{E}_{\text{ind}} = \frac{\vec{v}^2 \cdot Q \cdot \vec{E}}{c^2} - \frac{(\vec{v} \cdot Q \cdot \vec{E}) \cdot \vec{v}}{c^2} = \frac{\vec{v}^2 \cdot \vec{F}}{c^2} - \frac{(\vec{v} \cdot \vec{F}) \cdot \vec{v}}{c^2} \quad (48)$$

More precisely:

$$\vec{F}_{\text{ind}} = \frac{(\vec{v}_2 - \vec{v}_1)^2 \cdot \vec{F}}{c^2} - \frac{((\vec{v}_2 - \vec{v}_1) \cdot \vec{F}) \cdot (\vec{v}_2 - \vec{v}_1)}{c^2} \quad (49)$$

This formula can be applied even to the dipole. It shows that rotating disk does not dissipate energy.

Regarding formula (48) the power that the force performs pushing the body with velocity $\vec{v}$ is given by the following formula:

$$P = \vec{F} \cdot \vec{v} = \frac{\vec{v}^2 \cdot (\vec{v} \cdot \vec{F})}{c^2} - \frac{(\vec{v} \cdot \vec{F}) \cdot \vec{v}^2}{c^2} = 0 \quad (50)$$

This is the case of the superconductive ring. It does not dissipate energy because it conserves it. This means that formulas (20) and (31) are precise enough for ordinary usage and macroscopic physical assumptions.

These are all valid under only one small presumption: magnetic and electric fields are static non-moveable ones.

CORRECTION OF MAXWELL EQUATIONS

If we accept that equation (12) has wrong sign, than pair of Maxwell equations becomes:

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \quad (51)$$

And:

$$c^2 \cdot \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \quad (52)$$

Lets $\vec{K}$ is defined with:

$$\vec{K} = c \cdot \vec{B} \quad (53)$$

After equation (53) is involved into (51) and (52) it is obtained:

$$c \cdot \vec{\nabla} \times \vec{E} = \frac{\partial \vec{K}}{\partial t} \quad (54)$$

And:

$$c \cdot \vec{\nabla} \times \vec{K} = \frac{\partial \vec{E}}{\partial t} \quad (55)$$

Energetic equalization based on $\vec{K}$ transformation yields above symmetrical set of equation. If equation (55) is multiplied with imaginary unit i lead us to the following equation:

$$i \cdot c \cdot \vec{\nabla} \times \vec{K} = i \cdot \frac{\partial \vec{E}}{\partial t} \quad (56)$$

Let we define a new complex electromagnetic field as:

$$\vec{M} = \vec{E} + i \cdot \vec{K} \quad (57)$$

After addition (54) and (56) it is obtained:

$$\mathbf{c} \cdot \vec{\nabla} \times (\vec{\mathbf{E}} + i \cdot \vec{\mathbf{K}}) = \frac{\partial \vec{\mathbf{E}}}{\partial t} + i \cdot \frac{\partial \vec{\mathbf{K}}}{\partial t} \quad (58)$$

$\Rightarrow$

$$i \cdot \mathbf{c} \cdot \vec{\nabla} \times (\vec{\mathbf{E}} + i \cdot \vec{\mathbf{K}}) = i \cdot \frac{\partial \vec{\mathbf{E}}}{\partial t} - \frac{\partial \vec{\mathbf{K}}}{\partial t} \quad (59)$$

$\Rightarrow$

$$i \cdot \mathbf{c} \cdot \vec{\nabla} \times (\vec{\mathbf{E}} + i \cdot \vec{\mathbf{K}}) = - \left(\frac{\partial \vec{\mathbf{E}}}{\partial t} + i \cdot \frac{\partial \vec{\mathbf{K}}}{\partial t} \right) \quad (60)$$

Regarding equation (57) we have finally:

$$\boxed{i \cdot \mathbf{c} \cdot \vec{\nabla} \times \vec{\mathbf{M}} = - \frac{\partial \vec{\mathbf{M}}}{\partial t}} \quad (61)$$

This is equation of complex electromagnetic field. Regarding present Electromagnetic theory energy density is given with the following formulas:

$$\rho_{w_{\vec{\mathbf{E}}}} = \frac{\varepsilon}{2} \cdot \vec{\mathbf{E}}^2 \quad (62)$$

And:

$$\rho_{w_{\vec{\mathbf{K}}}} = \frac{\vec{\mathbf{B}}^2}{2 \cdot \mu} = \frac{\varepsilon \cdot \mathbf{c}^2 \cdot \vec{\mathbf{B}}^2}{2} = \frac{\varepsilon}{2} \cdot \vec{\mathbf{K}}^2 \quad (63)$$

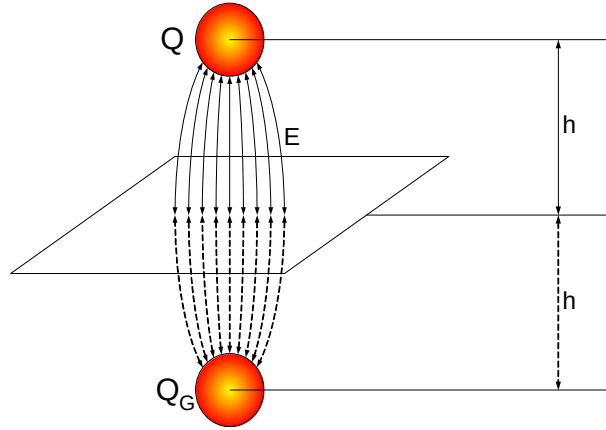
Formulas (62) and (63) are equal because energy density is same for both fields. Now it is obvious that these ones belong to unique single complex field. Regarding (57), (62) and (63) it can be concluded that is:

$$\rho_{w_{\vec{\mathbf{M}}}} = \frac{\varepsilon}{2} \cdot \vec{\mathbf{M}} \cdot \vec{\mathbf{M}}^* \quad (64)$$

Whereas $\vec{\mathbf{M}}^* = \text{CONJ}(\vec{\mathbf{M}})$.

ENERGY DENSITY AND THE IMAGE ELECTROSTATIC THEORY

Let we check whether the energy density equation (62) for electric field is completely correct regarding the electrostatic image theory. Let we imagine a charge over infinite conductive plain on altitude h, as it is shown on the picture below:



Formula for force between charge Q and plan regarding electrostatic image theory is given by the following formula:

$$F = \frac{1}{4 \cdot \pi \cdot \epsilon} \cdot \frac{Q^2}{(2 \cdot h)^2} = \frac{Q^2}{16 \cdot \pi \cdot h^2} \quad (65)$$

Formula can be obtained also by the formula of energy density as electrostatic pressure on the plain:

$$d\vec{F} = P \cdot d\vec{S} = U_w \cdot d\vec{S} = \frac{\epsilon \cdot \vec{E}^2}{2} \cdot d\vec{S} = \frac{\epsilon}{2} \cdot \frac{Q^2}{(4 \cdot \pi \cdot \epsilon)^2} \cdot \frac{d\vec{S}}{d^4} \quad (66)$$

Whereas:

$$d = \sqrt{h^2 + x^2 + y^2} \quad (67)$$

And:

$$d\vec{S} = d\vec{x} \times d\vec{y} \quad (68)$$

Final formula for force computation is:

$$F = \frac{\epsilon}{2} \cdot \frac{Q^2}{(4 \cdot \pi \cdot \epsilon)^2} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx \cdot dy}{(h^2 + x^2 + y^2)^2} = \frac{\epsilon}{2} \cdot \frac{Q^2}{(4 \cdot \pi \cdot \epsilon)^2} \cdot \frac{\pi}{r^2} = \frac{Q^2}{32 \cdot \pi \cdot \epsilon \cdot r^2} \quad (69)$$

There is obvious discrepancy between equation (65) and (69) because equation (65) yields twice bigger result than equation (69).

For concordance between electrostatic image theory and electrostatic energy density theory equation (62) shall be:

$$\boxed{\rho_{w_{\vec{E}}} = \epsilon \cdot \vec{E}^2} \quad (70)$$

CONCLUSION

One of equations (1), (12) or (20) must have wrong sign. Only equation that is more abstract than other ones in the set is Maxwell equation, i.e. equation (12). The other two ones deal with macroscopic phenomena, and it is small chance for equation (1) or (20) to have wrong sign because it should be already noticed in numerous experiments done all over the world in past century.

Equation (62) could be wrong too. It is small chance that electrostatic image theory is wrong and an experimental verification that has to show which theory is correct is so simply that there is no reason for words to waste on further discussion. Personally I believe that equation (62) is wrong although Nikola Tesla⁶ believed that electrostatic image theory is wrong and that grounded antenna actually does not have its image in the earth but that it produces terrestrial currents instead of image forming. He claimed that he proved that in his laboratory in Colorado Springs in series of experiments performed during 1899-1900.

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⁶ Nika Tesla, 1856-1943