Force in Hertzian Electrodynamics

Introduction

Hertz's field equations, which are first-order invariant under the Galilean transformation, are formally the same as Maxwell's covariant equations, except that every appearance of the non-invariant operator $\partial / \partial t$ in the latter is replaced by the first-order invariant total time derivative d / dt. The simplest expression of the latter is

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \left(\vec{\mathbf{v}}_d \cdot \vec{\nabla}\right). \tag{1}$$

The "convective" parameter $\vec{\mathbf{v}}_d$ appearing here may be interpreted as field detector (or sensor, radiation absorber, etc.) velocity with respect to the observer's inertial frame. Using a subscript "*Hz*" to denote Hertzian field quantities, the vacuum field equations (in Gaussian units) are thus

$$\nabla \times \vec{\mathbf{B}}_{Hz} - \frac{1}{c} \cdot \frac{d\vec{\mathbf{E}}_{Hz}}{dt} = \frac{4 \cdot \pi}{c} \cdot \vec{\mathbf{j}}_m \tag{2a}$$

$$\nabla \times \vec{\mathbf{E}}_{Hz} + \frac{1}{c} \cdot \frac{d\vec{\mathbf{B}}_{Hz}}{dt} = 0 \tag{2b}$$

$$\nabla \cdot \vec{\mathbf{B}}_{Hz} = 0 \tag{2c}$$

$$\nabla \cdot \vec{\mathbf{E}}_{Hz} = 4 \cdot \pi \cdot \rho \, , \tag{2d}$$

where $\vec{\mathbf{j}}_m = \vec{\mathbf{j}}_s + \vec{\mathbf{j}}_d$ is a "measured" source current; *i.e.*, measured by a source-current detector that co-moves with the field detector. Thus $\vec{\mathbf{j}}_m$ is the Maxwell source current $\vec{\mathbf{j}}_s$ (valid when $\vec{\mathbf{j}}_d \equiv -\rho \cdot \vec{\mathbf{v}}_d = 0$, the Maxwell case in which the field detector is at rest at the observer's field point), corrected for detector motion, $\vec{\mathbf{j}}_d \neq 0$. Invariance is manifested by the fact that Eqs. (2) are unaltered and

$$\vec{\mathbf{E}}_{Hz}' = \vec{\mathbf{E}}_{Hz} \quad \text{and} \quad \vec{\mathbf{B}}_{Hz}' = \vec{\mathbf{B}}_{Hz} \tag{3}$$

under the Galilean transformation,

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{v}} \cdot t \, , \qquad t' = t \, , \tag{4}$$

where $\vec{\mathbf{v}}$ is a relative velocity of inertial frames unrelated to $\vec{\mathbf{v}}_d$. Proofs of these invariances, including $\vec{\nabla}' = \vec{\nabla}$, $(d/dt)' = d/dt$, etc., have been given elsewhere [1-3]. Although aiming for the greatest generality would require us to allow arbitrary detector motions, we shall here simplify the discussion by considering $\vec{\mathbf{v}}_d$ to be constant (to an adequate approximation). That allows us to use Eq. (1) with assurance; otherwise, if the action of $\vec{\nabla}$ on $\vec{\mathbf{v}}_d$ were non-zero, extra terms would appear in many of the equations to be discussed.

Analogously to

$$\vec{\mathbf{B}}_{Maxwell} = \vec{\nabla} \times \vec{\mathbf{A}}_{Maxwell} \, , \tag{5a}$$

Eq. (2c) allows us to introduce a Hertzian vector potential via

$$\vec{\mathbf{B}}_{Hz} = \vec{\nabla} \times \vec{\mathbf{A}}_{Hz} \, , \tag{5b}$$

Eq. (2b) then yields

$$\nabla\times\vec{\mathbf{E}}_{Hz}+\frac{1}{c}\cdot\frac{d}{dt}\vec{\nabla}\times\vec{\mathbf{A}}_{Hz}=\vec{\nabla}\times\left(\vec{\mathbf{E}}_{Hz}+\frac{1}{c}\cdot\frac{d\vec{\mathbf{A}}_{Hz}}{dt}\right)=0\,. \tag{6}$$

Any quantity having a vanishing curl can be represented as the gradient of a scalar function, so $\vec{\mathbf{E}}_{Hz}+\frac{1}{c}\cdot\frac{d\vec{\mathbf{A}}_{Hz}}{dt}=-\vec{\nabla}\phi_{Hz}$. Thus the Hertzian electric field is related to potentials by

$$\vec{\mathbf{E}}_{Hz}=-\vec{\nabla}\phi_{Hz}-\frac{1}{c}\cdot\frac{d\vec{\mathbf{A}}_{Hz}}{dt}\,, \tag{7}$$

an invariant expression for $\vec{\mathbf{E}}_{Hz}$, provided that in Hertzian theory

$$\phi_{Hz}'=\phi_{Hz} \qquad \text{and} \qquad \vec{\mathbf{A}}_{Hz}'=\vec{\mathbf{A}}_{Hz}\,. \tag{8}$$

In Maxwell's theory we know that the definition of electric field is *force per unit charge*. Thus the force on charge q is $q\cdot\vec{\mathbf{E}}_{Maxwell}$. This, however, is only the electric force, seen as a partial force. The full physical force on the charge is not given directly by any solution of the field equations but must be separately postulated as the "Lorentz force,"

$$\vec{\mathbf{F}}_{Lorentz}=q\cdot\left(\vec{\mathbf{E}}_{Maxwell}+\vec{\mathbf{u}}\times\vec{\mathbf{B}}_{Maxwell}\right), \tag{9a}$$

where

$$\vec{\mathbf{E}}_{Maxwell}=-\vec{\nabla}\phi_{Maxwell}-\frac{1}{c}\cdot\frac{\partial\vec{\mathbf{A}}_{Maxwell}}{\partial t} \tag{9b}$$

and where $\vec{\mathbf{u}}$ is the velocity of charge q with respect to the observer's inertial frame. That Maxwell's field equations, while defining field quantities in terms of force on electric charge, fail to tell us anything directly about the *whole* physical force on an electric charge may be viewed as a flaw. The necessity in Maxwell's theory to introduce an extra "force postulate" is certainly not impressive evidence of logical economy. It creates a gulf between electromagnetism and electrodynamics that might be judged aesthetically displeasing. Let us see whether the Hertzian invariant formulation of electromagnetism can do better.

Hertzian Force

Since the electric field is traditionally *defined* as the force on unit electric charge, it seems anomalous in Maxwell's theory to say instead that such force involves something additional (*viz*., the magnetic component of the Lorentz force). If in Hertzian theory the whole of electrodynamics is to be contained in electromagnetism, the simplest realization of this objective requires that the full physical force on purely-electric charge q be expressible in terms of the purely-electric field solution of the field equations,

$$\vec{\mathbf{F}}_{Hz}=q\cdot\vec{\mathbf{E}}_{Hz}\,. \tag{10}$$

To say that the force on q is something else is inconsistent both with the above verbal definition of electric field and with logical economy. Since $\vec{\mathbf{E}}_{Maxwell}$ and $\vec{\mathbf{E}}_{Hz}$ are solutions of differently-parameterized field equations, the possible validity of (10) cannot be ruled out *a priori*. Let us therefore examine how such a "Hertzian" force is related to the Lorentz force. From (7) we see that (10) implies

$$\vec{\mathbf{F}}_{Hz} = q \cdot \left(-\vec{\nabla}\phi_{Hz} - \frac{1}{c} \cdot \frac{d\vec{\mathbf{A}}_{Hz}}{dt} \right), \tag{11}$$

wherein the potentials are source-related and presumably differ from those of Maxwell's theory at most by a gauge transformation. From Eq. (1) we have

$$\vec{\mathbf{F}}_{Hz} = q \cdot \left[-\vec{\nabla}\phi_{Hz} - \frac{1}{c} \cdot \left(\frac{\partial \vec{\mathbf{A}}_{Hz}}{\partial t} + \left(\vec{\mathbf{v}}_d \cdot \vec{\nabla} \right) \vec{\mathbf{A}}_{Hz} \right) \right]. \tag{12}$$

The vector identity [4]

$$\vec{\nabla}\left(\vec{\mathbf{v}} \cdot \vec{\mathbf{A}}\right) = \left(\vec{\mathbf{v}} \cdot \vec{\nabla}\right)\vec{\mathbf{A}} + \left(\vec{\mathbf{A}} \cdot \vec{\nabla}\right)\vec{\mathbf{v}} + \vec{\mathbf{A}} \times \left(\vec{\nabla} \times \vec{\mathbf{v}}\right) + \vec{\mathbf{v}} \times \left(\vec{\nabla} \times \vec{\mathbf{A}}\right)$$

reduces in our present special case of constant $\vec{\mathbf{v}} = \vec{\mathbf{v}}_d$, with $\vec{\mathbf{A}} \to \vec{\mathbf{A}}_{Hz}$, to

$$-\left(\vec{\mathbf{v}}_d \cdot \vec{\nabla}\right)\vec{\mathbf{A}}_{Hz} = \vec{\mathbf{v}}_d \times \left(\vec{\nabla} \times \vec{\mathbf{A}}_{Hz}\right) - \vec{\nabla}\left(\vec{\mathbf{v}}_d \cdot \vec{\mathbf{A}}_{Hz}\right). \tag{13}$$

Inserting this in (12), we have

$$\vec{\mathbf{F}}_{Hz} = q \cdot \left[-\vec{\nabla}\phi_{Hz} - \frac{1}{c} \cdot \frac{\partial \vec{\mathbf{A}}_{Hz}}{\partial t} + \frac{1}{c} \cdot \left(\vec{\mathbf{v}}_d \times \left(\vec{\nabla} \times \vec{\mathbf{A}}_{Hz}\right) - \vec{\nabla}\left(\vec{\mathbf{v}}_d \cdot \vec{\mathbf{A}}_{Hz}\right) \right) \right]. \tag{14}$$

Since we can identify our "field detector" (which has velocity $\vec{\mathbf{v}}_d$ in the laboratory inertial system) with the charge q (which has similarly-defined velocity $\vec{\mathbf{u}}$ appearing in Eq. (9a)), it is clear that $\vec{\mathbf{v}}_d = \vec{\mathbf{u}}$ holds. From this we see that (14) is beginning to acquire the appearance of the Lorentz force, Eq. (9), but with an extra force term

$$\vec{\mathbf{F}}_{Extra} = -\frac{q}{c} \cdot \vec{\nabla}\left(\vec{\mathbf{v}}_d \cdot \vec{\mathbf{A}}_{Hz}\right). \tag{15}$$

To find out more about this, let us subject (14) to a gauge transformation,

$$\phi_{Hz} \to \phi_{Hz} - \frac{1}{c} \cdot \frac{\partial \Lambda_{Hz}}{\partial t}, \qquad \vec{\mathbf{A}}_{Hz} \to \vec{\mathbf{A}}_{Hz} + \vec{\nabla}\Lambda_{Hz}, \tag{16}$$

where $\Lambda_{Hz} = \Lambda_{Hz}\left(x, y, z, t\right)$ is an arbitrary "gauge" function. When these substitutions are made in (14), that equation reduces to

$$\vec{\mathbf{F}}_{Hz} = q \cdot \left[-\vec{\nabla}\phi_{Hz} - \frac{1}{c} \cdot \frac{\partial \vec{\mathbf{A}}_{Hz}}{\partial t} + \frac{1}{c} \cdot \left(\vec{\mathbf{v}}_d \times \left(\vec{\nabla} \times \vec{\mathbf{A}}_{Hz}\right)\right) - \frac{1}{c} \cdot \vec{\nabla}\left(\vec{\mathbf{v}}_d \cdot \vec{\mathbf{A}}_{Hz}\right) - \frac{1}{c} \cdot \vec{\nabla}\left(\vec{\mathbf{v}}_d \cdot \vec{\nabla}\Lambda_{Hz}\right) \right]. \tag{17}$$

From the fact that Λ_{Hz} has not canceled from (17) we see that the extra force term (15) breaks the gauge invariance or "gauge symmetry" that characterizes Maxwell's theory. This means that in Hertz's theory gauge is no longer arbitrary, but has a physical significance. In order to restore an appearance of gauge symmetry by eliminating the extra force term (15), we see by inspection of (17) that a natural choice of Λ_{Hz} is

$$\vec{\nabla}\Lambda_{Hz} = -\vec{\mathbf{A}}_{Hz}. \tag{18}$$

This evaluates Λ_{Hz} as a line integral of the magnitude of $\vec{\mathbf{A}}_{Hz}$ along the (reversed) direction of that vector,

$$\Lambda_{Hz} = -\int \vec{\mathbf{A}}_{Hz} \cdot d\vec{\mathbf{s}}, \tag{19}$$

and leaves the gauge indeterminate within an additive constant. The consequence of this gauge choice is a physical force of the form

$$\vec{\mathbf{F}}_{Hz} = q \cdot \left[-\vec{\nabla}\phi_{Hz} - \frac{1}{c} \cdot \frac{\partial \vec{\mathbf{A}}_{Hz}}{\partial t} + \frac{1}{c} \cdot \left(\vec{\mathbf{v}}_d \times \left(\vec{\nabla} \times \vec{\mathbf{A}}_{Hz} \right) \right) \right], \tag{20a}$$

or, with (5b),

$$\vec{\mathbf{F}}_{Hz} = q \cdot \left[-\vec{\nabla}\phi_{Hz} - \frac{1}{c} \cdot \frac{\partial \vec{\mathbf{A}}_{Hz}}{\partial t} + \frac{1}{c} \cdot \left(\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}_{Hz} \right) \right], \tag{20b}$$

which has the form of the familiar Lorentz force law, Eq. (9), with the Hertzian potential quantities formally appearing for the Maxwellian ones. Thus by a suitable gauge choice it is possible to relate the Hertzian field quantities to observable force on a charge *q* by the same formal law as that traditionally employed to accomplish the same thing in Maxwell's theory. It is reasonable that this should be possible, since the Hertzian theory is a covering theory of the Maxwellian, in the sense that in the special case $\vec{\mathbf{v}}_d \to 0$ we have $\phi_{Hz} \to \phi_{Maxwell}$ and $\vec{\mathbf{A}}_{Hz} \to \vec{\mathbf{A}}_{Maxwell}$, and similarly for the field quantities.

It remains an open question – which we must leave unresolved here – whether the gauge choice that accomplishes this elimination of the "extra" force (15) is the physically valid one. That is, Hertzian theory is not fundamentally either gauge-symmetrical or space-time symmetrical, and thus leaves open the possibility that the observable physical force on a charge might differ from the Lorentz force by a term proportional to $\vec{\mathbf{v}}_d$. In other words, some experiment might conceivably be found that would render the alleged extra force term (or some other gauge-dependent force effect) observable.

Concerning the possibility of an extra force of the form (15), corresponding to the gauge choice $\Lambda_{Hz} =$ *constant* or zero [*cf.* (17)], we observe that $\vec{\mathbf{F}}_{Extra}$ is the gradient of a scalar quantity. Such a gradient, when integrated around any closed circuit (equivalently to the integration of an exact differential), must yield zero. Hence the extra force could not manifest itself in any experiments employing currents flowing in closed circuits. Maxwell maintained that currents flow *only* in closed circuits … but this overlooks charge motions in plasmas and possibly in antennas. (This latter may be controversial, since some physicists view transmitting and receiving antennas together as constituting closed circuits.) In any case it would not be trivial to devise a crucial experiment to test the extra gradient term in (14), and we shall not attempt to do that here.

The important aspects of the Hertzian force law (14) are (*a*) that it is deduced directly from the Hertzian field equations, without additional postulation, (*b*) that it is not distinguishable by ordinary laboratory experiments from the empirically well-confirmed Lorentz force law for the total force on charge *q*, (*c*) that it can be reduced by a gauge transformation to a formal analog, Eq. (20), of the Lorentz law, and (*d*) that the Hertzian electric field, as given by Eq. (7), produces the full physical force on the charge, Eq. (10), in agreement with the simple physical interpretation (or definition) that $\vec{\mathbf{E}}_{Hz}$ is the *total* force on unit charge. In short, in Hertzian theory there is no fundamental distinction between electromagnetism and electrodynamics, because a *complete* law of force on electric charge is provided by the electric solution of the field equations. This electric $\vec{\mathbf{E}}_{Hz}$-field solution, through the $d\vec{\mathbf{A}}_{Hz}/dt$ of Eq. (7), automatically incorporates velocity-dependent (magnetic) effects. There is no separate "magnetic" force, except upon magnetic monopoles, which are not considered.

We note that in Eq. (14) two gradient terms appear. These can be combined to form what we might at first guess to be a "physical potential energy," denoted by V; *i.e.*,

$$V = q\cdot\left(\phi + \frac{1}{c}\cdot(\vec{\mathbf{v}}_d\cdot\vec{\mathbf{A}})\right). \quad (?) \tag{21}$$

In order to investigate this as a possible physical potential energy, it will be useful to take a brief excursion into the territory of the Lagrangian method.

Lagrangian Formalism

From the well-known theory of the Lagrangian method [5] we shall borrow only two relations, first

$$L = T - U\,, \tag{22}$$

which defines the Lagrangian function L in terms of kinetic energy T and a "generalized" Lagrangian potential energy U, which in the case of velocity-dependent potentials generally differs from the physical potential energy V. Secondly, the Legendre transformation,

$$H = T + V = \sum\nolimits_k \dot{q}_k\cdot\frac{\partial L}{\partial\dot{q}_k} - L\,, \tag{23}$$

where H is the Hamiltonian, here interpreted as the total physical energy, and the $\dot{q}_k$ are Lagrangian generalized velocity components. Since U is non-physical (in the case of velocity-dependent potentials), its determination is something of a guessing game. Let us guess a scalar function of the form

$$U = q\cdot\left(\phi + \frac{k}{c}\cdot\left(\vec{\mathbf{v}}_d\cdot\vec{\mathbf{A}}\right)\right), \tag{24}$$

where k is an undetermined constant. We may simplify to a one-dimensional equivalent problem by introducing a scalar coordinate r aligned along the direction of the vector $\vec{\mathbf{v}}_d$, so that $\vec{\mathbf{v}}_d$ has the magnitude $\dot{r}$. This allows us to make the replacement

$$(\vec{\mathbf{v}}_d\cdot\vec{\mathbf{A}}) \leftrightarrow A_r\cdot\dot{r}\,, \tag{25}$$

where A_r does not depend on $\dot{r}$. Confining attention to the non-relativistic case, we have $T = (1/2)\cdot m\cdot\dot{r}^2$, where m is the mass of the charge q that we consider to act as our "field detector." Then from (22), (24), and (25)

$$L = (1/2)\cdot m\cdot\dot{r}^2 - q\cdot\left(\phi + \frac{k}{c}\cdot A_r\cdot\dot{r}\right), \tag{26}$$

whence

$$\frac{\partial L}{\partial\dot{r}} = m\cdot\dot{r} - \frac{k\cdot q}{c}\cdot A_r\,. \tag{27}$$

From (23), written as $H = \dot{r}\cdot\frac{\partial L}{\partial\dot{r}} - L$, with the help of (26) and (27), we get

$$H = m\cdot\dot{r}^2 - \frac{k\cdot q}{c}\cdot A_r\cdot\dot{r} - (1/2)\cdot m\cdot\dot{r}^2 + q\cdot\phi + \frac{k\cdot q}{c}\cdot A_r\cdot\dot{r}$$

or

$$H = T + V = (1/2)\cdot m\cdot \dot{r}^2 + q\cdot\phi\,. \tag{28}$$

Notice that the k-terms have canceled; so k can be assigned any value, including zero. Thus our guess embodied in Eq. (21), that the physical potential energy V (defined as that which enters the Hamiltonian) might contain a term proportional to $\left(\vec{\mathbf{v}}_d\cdot\vec{\mathbf{A}}\right)$ is disproven. Eq. (28) indicates that, regardless of whether such a term is present in or absent from the Lagrangian generalized potential energy U, the physical potential energy V can only be

$$V = q\cdot\phi\,. \tag{29}$$

The customary Lagrangian derivation [5] of the Lorentz force law assumes $k = -1$. An attempt might be made to justify this choice by the following argument, which appeals to physics rather than to any intrinsic feature of the Lagrangian formalism: The quantity in Eq. (27) is recognized as the Lagrangian generalized or "canonical" momentum,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\cdot\dot{r} - \frac{k\cdot q}{c}\cdot A_r\,,$$

and the term in A_r is identified as the physical momentum of the electromagnetic field. With the choice $k = -1$ this field momentum may be considered to add to the mechanical momentum $m\cdot\dot{r}$, so

$$p_r = m\cdot\dot{r} + \left(q/c\right)\cdot A_r$$

is validated, apparently, by endowing the canonical momentum p_r with a physical significance as total momentum of field plus particle. However, since the Lagrangian generalized potential energy U is nonphysical, one is hardly prepared to learn that the corresponding Lagrangian generalized ("canonical") momentum p_r is physical. In fact, if one learns this, one then has to unlearn it immediately, because in classical mechanical theory (both relativistic and non-relativistic) [5], and in Dirac's electron theory, it is actually $m\dot{r}$ that is treated as physical – since, to describe the advent of a field, it is $m\cdot\dot{r} = p_r - \frac{q}{c}\cdot A_r$ that is substituted for p_r in the field-free expression for total energy. That is, field momentum is subtracted from (not added to) canonical momentum to get physical momentum of the particle.

All that can be said, finally, concerning (21) is that it was a bad guess about the physics. In view of (29) and the assumption that H is total physical energy, it would appear that the extra term in $\left(\vec{\mathbf{v}}_d\cdot\vec{\mathbf{A}}\right)$ cannot be part of a physical potential energy. Does this mean that the extra force term in our "Hertzian" force law (15) is non-physical? If so, it can be disregarded – and in that case the Hertz and Lorentz force laws become formally identical. We have already noted that this alleged extra force term would be difficult to test by laboratory experiments (impossible by use of currents flowing in closed circuits). If the term is indeed non-physical, as our failed attempt to incorporate it in a physical potential energy would indicate, then *in principle* no experiment could reveal it. The foregoing considerations may be viewed as constituting an independent quasi-physical argument in favor of the gauge choice (18), which eliminates entirely the extra force term (15) in Hertzian theory.

We are assuming that H in electromagnetic theory represents total observable energy. What can be said about this? With velocity-independent potentials the Lagrange method is known to be reliable. But there is no question – when one deals with velocity-dependent potentials – that the U-function of the Lagrange method is non-physical and thus amounts to a contrivance – as does the Lagrangian itself. It seems evident that the canonical momentum is also not the physical momentum, since it includes a "field momentum" term that does not survive to make a contribution to the total physical energy (28). Why is field momentum absent from total energy? This may be the case because it is permissible to think of the force-exerting field agent, the photon, either as "virtual" – in which case it manifests no physical attributes – or as possessed of no degrees of freedom independent of matter – in which case it can affect the momentum of its source and of its sink, but not of intermediate space. In accord with the spirit of quantum mechanics, there is no way to capture or reify this alleged momentum *while* it is acting across space, except to replace "space" with a detector. And if L is a non-physical contrivance and H does *not* represent total physical energy, what has become of the physics in all this carnival of formalism? It would seem best to hold onto H as total energy and to revise our last-century picture of electromagnetic force as "propagating" – *i.e.*, as energy flying (locally and causally) through space – such a picture being a metaphysical imposition at odds with all else we have learned about quantum processes.

It is worth mentioning that in Eq. (6-31) of his book [5] Goldstein expresses the Lorentz force in a form equivalent to

$$\vec{\mathbf{F}}_{Lorentz} = q \cdot \left(-\vec{\nabla}\phi - \frac{1}{c} \cdot \frac{d\vec{\mathbf{A}}}{dt} + \frac{1}{c} \cdot \vec{\nabla}\left(\vec{\mathbf{v}} \cdot \vec{\mathbf{A}}\right) \right). \tag{30}$$

Thus, had he not been blinded by the science of covariance he might have discovered the science of invariance, simply by recognizing the closed-circuit unobservability of the last gradient term … for with its elimination Eq. (30) reduces to our presently proposed Eq. (11), the Hertzian invariant form.

We must leave the subject here with the tentative conclusion that the Hertzian and Lorentz force laws are *probably* for all observational purposes physically equivalent. If that is *not* correct, then experiment must be able to decide whether the Hertzian Eqs. (11) and (14) should be modified by addition of the final gradient term in (30) … which is the same question as whether Eq. (30) should be modified by omission of its last term. By one way of looking at it, the physicality of the Lorentz force law depends on the observability of the last term in (30) – for, failing such observability, it would become physically permissible to omit that term entirely – in which case the Hertzian and Lorentz force laws would become not merely predictively equivalent but formally identical.

Hertzian vs. Maxwellian Fields

To those reared on covariance, the invariance claimed in Eq. (3) may seem counter to known *fact*. Thus the field "scramblings" (whereby electric fields can "become" magnetic fields or linear combinations of both kinds of fields) asserted by covariance are today widely accepted as an inherent feature of the field; *i.e.*, a fact about the underlying physics. But what substantiates this "fact" is not the changing of one kind of field into another … what changes in every case is the state of motion of the field detector. The field itself, conceived as *ding an sich* – something "out there" that is *independent of the state of motion of its detector* – does not exist. (This does not imply that there is "nothing" out there … no ontological profundity is intended.) When we say that one inertial observer "sees" one kind of field, and another inertial observer sees another kind, what we are in fact saying is that field detectors in different states of motion (each co-moving with a different inertial observer) detect different mixes, or covariantly-related scramblings, of components of the field at a common "field point" that is momentarily shared in space and time. The two observers disagree about what the field "is" at that point, because they are measuring the "same field" with differently-moving instruments. In a sense, each instrument may be said to create its own field. (This is a variant of Bohr's famous dictum, "The apparatus as a whole makes the measurement.") However, if instead we consider a given state of motion, defined by that of a *single* field-detection instrument, then all observers must agree on the readings of that chosen instrument; so in this case there is no ambiguity or multi-valuedness. We can then speak of an observer-independent uniqueness or invariance of the field.

In order to bring out explicitly and quantitatively this invariant aspect of field description it is necessary to abandon covariant (Maxwellian) field equations and substitute invariant ones. This is what Hertz's mathematics did at the first order, to which we confine attention here. To recapitulate: In Maxwell-Einstein physics each inertial observer is equipped with his own "private" field detector, permanently at rest at his own co-moving field point. When the field points of two such relatively-moving observers momentarily coincide – implying collision of their instruments – the measured field components, as displayed by the two instruments, are two sets of numbers related covariantly and quantified by Maxwell's field equations, considered valid in both inertial systems. (It is this dual validity that underpins the constancy of "*c*" in all inertial systems – Einstein's second postulate.)

By contrast, in Hertzian physics there is only one "public" field detector under consideration, and there are any number of observers, who need not be inertial – but may for convenience be considered so. In order to quantify, by Hertz's field equations, the readings at any moment of that unique "public" instrument (idealized as a point in space), we may picture a multiple coincidence of field points (co-moving with each of two or more inertial observers) with that instrument. (Such observer-instrument relative velocities are parameterized by $\vec{\mathbf{v}}_d$.) Then the various observers involved must at that event of multiple coincidence read from the display of that particular instrument the same numbers – for the simple reason that there are no other numbers there to be read. This is the (trivial) physical meaning of numerical invariance, which is reflected also in form invariance [*cf*. Eqs. (3) and (8), above]. Since the observers' field points are notional (mathematical points), their "collisions" with the instrument cause no physical disruption, such as would occur in the case of covariance (multiple instruments, each a composition of matter, in physical coincidence). Since Maxwell's equations are not involved, there is no universal constancy of *c*. All inertial observers honor the relativity principle by using the same field equations – but these are Hertz's, not Maxwell's. Hertz's equations lack spacetime symmetry and assert a formal effect upon light speed of detector velocity $\vec{\mathbf{v}}_d$. (This of course requires the development of a new kinematics [1], in conjunction with higher-order refinement of the Hertzian equations themselves [2,3].)

It will be apparent that two quite distinct ideas of "invariance," hence of "relativity" are involved. The Maxwell-Einstein idea is that for different inertial observers the "laws of nature" describing the field are the same because, when each observer is equipped with his own "private" field-measuring instrument and performs the same measurement operations, each by *replicating* the experimental procedures of the others will measure (when field points coincide) not the same numbers but sets of numbers covariantly related. Such a conception of what motional "relativity" is about evidently incorporates a physical assumption of replicability of experiments, whether in the same frame or in different frames, either simultaneously or sequentially.

The Hertzian idea is entirely different and in a sense more primitive. It is that the job of physics is to describe what is *out there* in nature (entirely apart from notional observers), on a one-time, one-place, one-occurrence basis. Such description must be invariant – that is, independent of state of motion of any observers. In the case of fields, the essential element that is "out there" is the absorber or field detector – that which "measures," or by localizing "creates," the field. Only a single instrument is involved in any (unique) episode of measurement, although multiple observers may be present. Replicability of experiments is *not* assumed (nor is it, at the strictly classical level, denied). The mathematics effectuating the Maxwell-Einstein view interprets "invariance" to mean covariance – the linear "scrambling" of field components – which by definition discards numerical invariance. The mathematics effectuating the Hertzian view demands true invariance, whereby all symbols appearing in the "laws of nature" [*e.g.*, the field equations, Eq. (2)] transform in place without altered relationships, as in Eqs. (3) and (8), so that *both* formal and numerical invariance are attained.

At the classical level of physical description there is no obvious basis for preferring one of these rival types of "relativity" and "invariance" over the other. But when we consider the quantum limit of measurement theory, or address the weak-field (one-quantum) limit of field theory, the superiority of the more primitive (or less assumption-laden) Hertzian version becomes at once evident. In a word, the Maxwell-Einstein field theory works only where there are many field quanta present – enough in principle that sufficient numbers are absorbed by each notional detector, co-moving with its own inertial observer, to iron-out statistical fluctuations. In that case the smoothed numbers displayed on each detector are covariantly related. But, where there are so few field quanta present that this smoothing ceases to be effective, covariance fails catastrophically. Thus if the field is so weak that only a single photon is present, and two macroscopic detection instruments compete for it, only one can "win." That is, only one of these instruments can successfully "measure" the field. The other must register zero – which cannot be covariantly related to anything. So, in the weak-field limit the model of a plurality of "private" measuring instruments (underlying the covariance-based conception of "relativity") fails – as does the assumption of replicability of their measurements.

The Hertzian type of relativity, based on invariance, does not fail in either weak-field or strong-field limit. When a single "public" instrument is present in a field of arbitrary strength, it must contend with simple statistical fluctuations (more severe as the field weakens), but in a straightforward way – without the complication of *competition* for field quanta with other macro instruments (notionally) present at the same place and time. There is no assumption of replicability of measurements made simultaneously at a given place by a plurality of macro instruments, such as underlies the Maxwell-Einstein picture of sets of measured field-component numbers covariantly related. The latter "classical" conception is counter-indicated by all that the twentieth century has taught us about the physics of the quantum world. To put it in a nutshell, covariance fails *prima* *facie* if there are more observers-cum-detectors present than field quanta – for in that case the competition is too fierce and some observers must fail to detect any field at all.

In sum: At high field intensities either the Maxwell-Einstein picture (covariance with many inertial-observers-cum-detectors) or the Hertz picture (invariance with many observers of a single detection instrument) will work. But in the low-field limit only the Hertz picture remains conceptually viable and compatible with quantum physics.

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