WHY TESLA WON THE WAR OF CURRENTS

Tesla won the Current War that he waged with Edison due to three major advancements: (1) Electric Transformer, (2) Three-phase Current power grid, and (3) Asynchronous Motor. Tesla was promoter of AC current while Edison was promoter of DC electrical power grid and only one system had to be globally adopted.

Let us pass trough all three major advancements of Three-phase Grid:

(1) ELECTRIC TRANSFORMER

DC current utilizes cable of power line 100% because current runs constantly all the time, and the power of loss trough electric line is:

$$\mathsf{P}_{\mathsf{loss}} = \mathsf{R}_{\mathsf{line}} \cdot \mathsf{I}_{\mathsf{line}}^{2} \tag{1}$$

Whereas P_{loss} is power of thermal loss in line's conductors, R_{line} is electric resistance of the line and I_{line} is current running trough the power line.

Useful power transmitted by the power line is:

$$P = (U_{DC} - R_{line} \cdot I_{line}) \cdot I_{line} = U_{DC} \cdot I_{line} - R_{line} \cdot I_{line}^{2} = U_{DC} \cdot I_{line} - P_{loss}$$
(2)

Above equation can be solved on I_{line} and inserted in (1) obtaining following equation:

$$P_{loss} = \frac{U_{DC}^{2} - U_{DC} \cdot \sqrt{U_{DC}^{2} - 4 \cdot P \cdot R}_{line} - 2 \cdot P \cdot R_{line}}{2 \cdot R_{line}}$$
(3)

Whereas P_{loss} is thermal loss in the electric line, P is useful power on the end of the line available to consumers, U_{DC} is electric potential between the ends of the line on its beginning and R_{line} is electric resistance of the line itself.

Above equation firmly states that we need to raise potential as high as possible on the beginning of power electric line to minimize thermal loss in the conductors across the line because by increasing of the potential we are simultaneously decreasing current that causes thermal loss in the power line itself. We can obtain this voltage augmentation by electric transformers in AC electric grid or by rotary converters in DC electric grid. Electric transformer is quite simple and durable device with extremely high efficiency, in those times usually higher than η =95% simultaneously without moving part at all (today efficiency is nearly 100% for transformers with amorphous core) while DC rotary transformer is able to transform only AC current and therefore for transformation of DC current only available solution in those times was big, inefficient and noisy DC rotary converter.

We should bear in mind that we need at least two such devices on every single power electric line – initial one to rise potential for transport trough the power line and terminal one to lower it down again to the level suitable for residential and industrial consumers, one on the beginning and one on the end of the power line.

Therefore, the efficiency of a pair of coupled AC converters was 90% ($0.9 \approx 0.95^2$) for AC line and only 56% for pair of couple DC converters ($0.56 \approx 0.75^2$) making AC grid much more efficient than DC one.

For the case of AC line that is driven by sinusoidal excitation the situation is a bit more complicated while potential varies in accordance with sinusoidal function:

$$U(t) = U_{\max} \cdot SIN(2 \cdot \pi \cdot f \cdot t) = U_{\max} \cdot SIN(\omega \cdot t)$$
(4)

So, we need to find the energy loss for a single period:

$$\mathsf{E} = \int_{0}^{\mathsf{T}} \mathsf{P}(t) \cdot \mathsf{d}t = \frac{\mathsf{U}_{\max}^{2}}{\mathsf{R}} \cdot \int_{0}^{\frac{1}{\mathsf{f}}} \mathsf{SIN}(2 \cdot \pi \cdot \mathsf{f} \cdot \mathsf{t})^{2} \cdot \mathsf{d}t = \frac{\mathsf{U}_{\max}^{2}}{2 \cdot \mathsf{R} \cdot \mathsf{f}}$$
(5)

Energy for the same amount of time equivalent to DC current with potential $U_{\text{eff}}\xspace$ is:

$$E = P \cdot T = \frac{U_{eff}^{2}}{R} \cdot \frac{1}{f} = \frac{U_{eff}^{2}}{R \cdot f}$$
(6)

We can compare these two electric lines by equalizing the energy of DC and AC currents:

$$\frac{U_{\text{max}}^{2}}{2 \cdot R \cdot f} = \frac{U_{\text{eff}}^{2}}{R \cdot f}$$
(7)

 \Rightarrow

$$U_{eff} = \frac{U_{max}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \cdot U_{max}$$
(8)

DC effective potential U_{eff} performs identical power on electric consumer as AC potential U_{max} on the same line, so if we replace AC potential U_{max} with DC potential U_{eff} then should be no difference in the power transmissions between these two lines.

We can calculate the same via average power over the period:

$$\overline{\mathsf{P}} = \frac{1}{\mathsf{T}} \cdot \int_{0}^{\mathsf{T}} \mathsf{U}(\mathsf{t}) \cdot \mathsf{d}\mathsf{t} = \mathsf{f} \cdot \int_{0}^{\frac{1}{\mathsf{f}}} \frac{\mathsf{U}_{\mathsf{AC}}(\mathsf{t})^{2}}{\mathsf{R}} \cdot \mathsf{d}\mathsf{t} = \frac{\mathsf{f} \cdot \mathsf{U}_{\mathsf{max}}^{2}}{\mathsf{R}} \cdot \int_{0}^{\frac{1}{\mathsf{f}}} \mathsf{SIN}(2 \cdot \pi \cdot \mathsf{f} \cdot \mathsf{t})^{2} \cdot \mathsf{d}\mathsf{t} = \frac{\mathsf{U}_{\mathsf{max}}^{2}}{2 \cdot \mathsf{R}}$$
(9)

Therefore, it is convenient to define the term of **eff**ective potential U_{eff} as the equivalent potential of DC current that causes the same heat on resistance R as corresponding AC current with maximal alternate potential U_{max} :

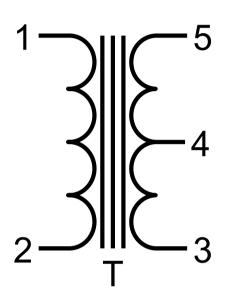
$$\overline{P} = \frac{U_{max}^{2}}{2 \cdot R} = \frac{U_{eff}^{2}}{R}$$
(10)

 \Rightarrow

$$U_{\rm eff} = \frac{U_{\rm max}}{\sqrt{2}} \tag{11}$$

Above equation states that mono-phase AC line has 50% less power throughput than DC line for the same maximal voltage which is huge disadvantage of mono-phase AC power line in respect to DC one. AC Electric transformer has a quite simple construction; it is consisted of at least two stationary coils winded around mutual ferromagnetic core while rotary convertor is essentially DC motor and DC generator fused in the single device sharing the same rotor with two sets of brushes and two armatures. These were quite remarkable devices in most subways around the world until recently. Nowadays we are using electronically switched transformers for transforming both AC and DC power lines. Symbol for electric transformer with two windings is:

Fig. 1



Operation of AC electric transformer is not clear at all. This simple, ugly and nasty device mocks professors all around the globe because it stubbornly disobeys to the Maxwell's equations. The voltage on the secondary winding is proportional to the voltage on the primary one instead to be proportional to the second time derivative of the current running trough primary winding as pair of Maxwell's equations firmly states it should be. The situation is additionally aggravated by the fact that paired Biot-Savart & Faraday equations claim the same:

$$\frac{U_1}{n_1} = \frac{U_2}{n_2}$$
(12)

Above equation seemingly defies common logic because primary coil acts as emissive antenna and secondary coil acts as receptive one, thus we may expect that increase of both antennas will increase transmitted voltage, but it is not the case with the real transformer. Actually, smaller emissive antenna causes higher voltage on the receptive one and bigger receptive antenna increases reception voltage as expected. Anyway, above equation is duly correct one regardless it is logic to us or not, firmly depicting operation of a real electric transformer currently in use all over the globe.

Electric transformer is so important for the proper understanding of our technical civilization and contemporary state of science because Maxwell's

equations was derived directly from the Faraday law of induction with aid of Green's vector identity, Lorentz transformation was derived directly from paired Maxwell's equations. Then Einstein SRT and GRT theories are subsequently derived from Lorentz transformation. There are no more than two pages of paper needed for proper derivation of all these aforementioned. We know today that Einstein cannot be dully correct due to the few paradoxes mentioned on my website like Twin paradox in which every single one may claim that they is only moving one, absolute inertial frame of centrifugal force and unclosed mechanical circuits in this theory. Assumed invariance of the inertial frame that cannot be physically plausible as it is shown for the case of inertia and centrifugal force, and unfounded equalization of inertial and gravitational masses are firmly disproved by experimental discovery of Higgs boson. And yet there is a question how Einstein can be false at all while there is relatively short path of preceding aforementioned equations with seemingly rock solid validity in each single step... However, there is a simple, also rock solid explanation too for all this mess embodied in two phenomena: Doppler's effect and retarded potentials. Both twins will see exactly as predicted by SRT, but this is rather caused by retardation of the electromagnetic field embodied in the light (or delaying of the seeing) then by any kind of sorcery including even relativistic one.

Detailed derivation of the electric transformer's operational model is too complicate exceeding the scope of this text, but certainly quite different from the official model in which flux in ferromagnetic core does not depend on the voltages between ends of several windings.

THREE-PHASE CURRENT ELECTRIC GRID

Equation (11) exposes huge disadvantage of mono-phase AC electric grid over DC one showing it requires 50% more copper for the same amount of thermal loss in DC electric power line for the same power. Tesla resolved this problem relying only on a simple trigonometric identity:

$$\sum_{k=0}^{n-1} SIN\left(\omega \cdot t + \frac{2 \cdot \pi}{n} \cdot k\right) = 0$$
(13)

For the case of three-phase current we have:

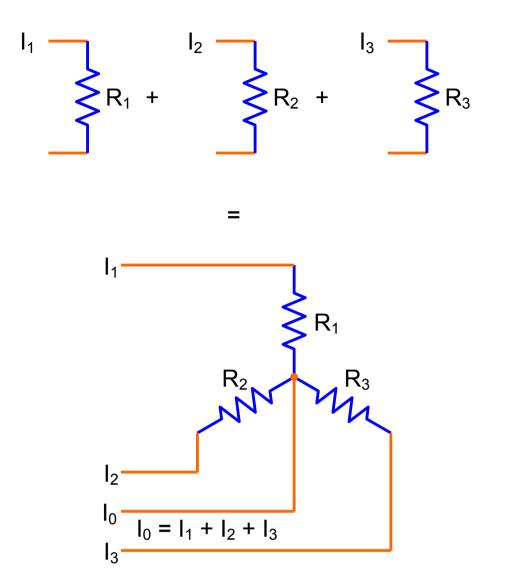
$$SIN(\omega \cdot t) + SIN\left(\omega \cdot t + \frac{2 \cdot \pi}{3}\right) + SIN\left(\omega \cdot t + \frac{4 \cdot \pi}{3}\right) = 0$$
(14)

Above equation has mathematically identical form that is more convenient:

$$SIN\left(\omega \cdot t - \frac{2 \cdot \pi}{3}\right) + SIN\left(\omega \cdot t\right) + SIN\left(\omega \cdot t + \frac{2 \cdot \pi}{3}\right) = 0$$
(15)

Or:

$$SIN(\omega \cdot t - 120^{\circ}) + SIN(\omega \cdot t) + SIN(\omega \cdot t + 120^{\circ}) = 0$$
(16)



This will be demonstrated by the following thought experiment:

Above figure depicts simple fusion of three electric consumers in single contact on one their ends, whereas AC current I₀ running trough null wire is sum of currents running trough supplying wires I₁, I₂ and I₃, all in accordance with Kirchhoff laws. But, if we shift phase of one supplying line for 120° (i.e. $\frac{2}{3} \cdot \pi$) and another phase also shift for 240° respectively, or better for -120° as mathematically it is the same, then we will obtain three-phase system depicting completely different situation:

Fig. 3

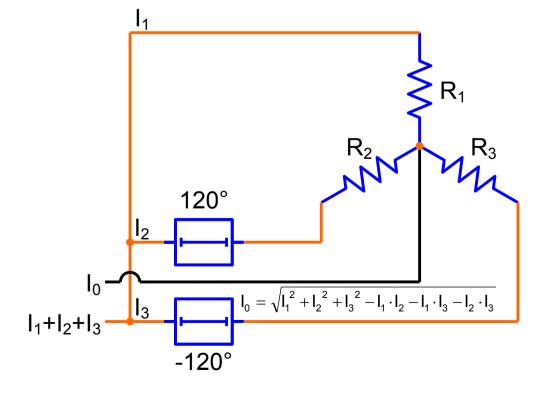


Fig. 2

Above picture portraits situation of true three-phase system and in this case of identical resistors the current running trough null wire I₀ is going to be trivial, i.e. equal to zero, according abovementioned mathematical trigonometric identity! Three-phase system halves amount of required copper in respect to mono-phase system simply because returning wires vanishes for identical consumers. We had that mono-phase AC system transmission's efficiency has half of DC system power for the same maximal voltage while for three-phase system transmission capacity fully matches the capacity of DC system retaining simplicity of voltage transformation by AC transformers. We always can balance large power electric grid by meticulous design to have nearly equal load on every phase due to sheer number of consumers and therefore it can be keeping null wire I_0 with negligible cross section in respect to the rest of three-phase wires cross sections. Huge advantage of three-phase AC system over the DC one is mainly based on ability of efficient and reliable AC transformer's usage. Seemingly, there is a logical paradox that we need three wires for AC line with same amount of copper as in case of only two wires in a DC line with identical amount of copper, which is clear demonstration of the practical and crude power of pure mathematics, if used properly.

Trigonometric formula for the sum of two harmonics with the same frequency ω and different phase angles θ_1 and θ_2 and amplitudes I_1 and I_2 is:

$$I_{1} \cdot SIN(\omega \cdot t + \theta_{1}) + I_{2} \cdot SIN(\omega \cdot t + \theta_{2}) =$$

$$= \sqrt{I_{1}^{2} + I_{2}^{2} + 2 \cdot I_{1} \cdot I_{2} \cdot COS(\theta_{1} - \theta_{2})} \cdot SIN\left(\omega \cdot t + ATAN\left(\frac{I_{1} \cdot SIN(\theta_{1}) + I_{2} \cdot SIN(\theta_{2})}{I_{1} \cdot COS(\theta_{1}) + I_{2} \cdot COS(\theta_{2})}\right)\right)$$

Following equation is quite important for finding effective electric potential between two irregular phases:

$$U_{1,2} = \sqrt{U_{0,1}^{2} + U_{0,2}^{2} - 2 \cdot U_{0,1} \cdot U_{0,2} \cdot COS(\theta_{1,2})}$$
(18)

(17)

With above equation emerges geometric connotation of joined harmonics with same frequency because above equation is Cosine Theorem for a triangle.

Irregular three-phase system is the one with angles between phases different than 120° or $\frac{2}{3} \pi$. Angle $\theta_{1,2}$ between phases U₁ and U₂ is:

$$COS(\theta_{1,2}) = \frac{U_{0,1}^{2} + U_{0,2}^{2} - U_{1,2}^{2}}{2 \cdot U_{0,1} \cdot U_{0,2}}$$
(19)

Above equation yields ability to determine angle between phases with pocket calculator and preferably True RMS voltmeter only, without involvement of any highly specialized and expensive measuring device, like portable two channels' oscilloscope.

Effective electric current is going to be:

$$I_{1,2} = \sqrt{I_1^2 + I_2^2 + 2 \cdot I_1 \cdot I_2 \cdot COS(\theta_{1,2})}$$
(20)

Effective current I_0 running trough null wire in the case of regular three-phase system with 120° angle between phases and with effective currents I_1 , I_2 and I_3 is:

$$I_{0} = \sqrt{I_{1}^{2} + I_{2}^{2} + I_{3}^{2} - I_{1} \cdot I_{2} - I_{1} \cdot I_{3} - I_{2} \cdot I_{3}}$$
(21)

Above equation can be derived on two ways, by Phasors' Method and by Classical Wave Trigonometric Addition method.

The derivation based on Phasors' Mehtod is an easy task:

$$I_{\text{complex1}} = I_1 \cdot (COS(0) + i \cdot SIN(0)) = I_1$$
(22)

$$I_{\text{complex2}} = I_2 \cdot \left(\text{COS}\left(\frac{2 \cdot \pi}{3}\right) + i \cdot \text{SIN}\left(\frac{2 \cdot \pi}{3}\right) \right) = I_2 \cdot \left(-\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2}\right)$$
(23)

$$I_{\text{complex3}} = I_3 \cdot \left(\text{COS}\left(-\frac{2 \cdot \pi}{3} \right) + i \cdot \text{SIN}\left(-\frac{2 \cdot \pi}{3} \right) \right) = I_3 \cdot \left(-\frac{1}{2} - i \cdot \frac{\sqrt{3}}{2} \right)$$
(24)

Whereas I_1 , I_2 and I_3 are effective values of currents, $I_{comple1}$, $I_{comple2}$ and $I_{comeplex3}$ are phasors, i.e. complex values of currents, $i^2 = -1$. According previous drawing it is:

$$I_{\text{complex0}} = I_{\text{complex1}} + I_{\text{complex2}} + I_{\text{complex3}}$$
(25)

By definition, the effective null current is:

$$\mathbf{I}_{0} = \left|\mathbf{I}_{\text{complex0}}\right| = \mathbf{Re}\left(\mathbf{I}_{\text{complex0}}\right)^{2} + \mathbf{Im}\left(\mathbf{I}_{\text{complex0}}\right)^{2}$$
(26)

 \Rightarrow

$$I_{0} = \sqrt{I_{1}^{2} + I_{2}^{2} + I_{3}^{2} - I_{1} \cdot I_{2} - I_{1} \cdot I_{3} - I_{2} \cdot I_{3}}$$
(27)

By definition the tangent of phase's angle is:

$$\mathsf{TAN}(\theta_0) = \frac{\mathsf{Im}(\mathsf{I}_{\mathsf{complex0}})}{\mathsf{Re}(\mathsf{I}_{\mathsf{complex0}})} = \frac{(\mathsf{I}_2 - \mathsf{I}_3) \cdot \sqrt{3}}{2 \cdot \mathsf{I}_1 - \mathsf{I}_2 - \mathsf{I}_3}$$
(28)

Another derivation based Classical Wave Trigonometric Addition method, i.e. composition of wave functions proceeds, waves of all three phases are:

$$I_{1}(t) = I_{1} \cdot SIN(\omega \cdot t)$$
(29)

$$I_{2}(t) = I_{2} \cdot SIN\left(\omega \cdot t + \frac{2 \cdot \pi}{3}\right)$$
(30)

$$I_{3}(t) = I_{3} \cdot SIN\left(\omega \cdot t - \frac{2 \cdot \pi}{3}\right)$$
(31)

Sum of these waves yields a wave running trough null wire:

$$I_{0}(t) = I_{1}(t) + I_{2}(t) + I_{3}(t)$$
(32)

Applying basic trigonometric decompositions, we have:

$$I_{0}(t) = \frac{2 \cdot I_{1} - I_{2} - I_{3}}{2} \cdot SIN(\omega \cdot t) + \frac{\sqrt{3} \cdot (I_{2} - I_{3})}{2} \cdot COS(\omega \cdot t)$$
(33)

We are going to rely on following trigonometric identity to simplify above equation:

$$A \cdot SIN(\phi) + B \cdot COS(\phi) = \sqrt{A^2 + B^2} \cdot SIN\left(\phi + ATAN\left(\frac{B}{A}\right)\right)$$
(34)

Then, following equation is obtained yielding both amplitude and phase:

$$I_{0}(t) = \sqrt{I_{1}^{2} + I_{2}^{2} + I_{3}^{2} - I_{1} \cdot I_{2} - I_{1} \cdot I_{3} - I_{2} \cdot I_{3}} \cdot SIN\left(\omega \cdot t + ATAN\left(\frac{(I_{2} - I_{3}) \cdot \sqrt{3}}{2 \cdot I_{1} - I_{2} - I_{3}}\right)\right)$$
(35)

Terms defining amplitude is easily recognizable in above equation and it is:

$$I_{0} = \sqrt{I_{1}^{2} + I_{2}^{2} + I_{3}^{2} - I_{1} \cdot I_{2} - I_{1} \cdot I_{3} - I_{2} \cdot I_{3}}$$
(36)

Phase's angle is:

$$\mathsf{TAN}(\theta_{0}) = \frac{(\mathsf{I}_{2} - \mathsf{I}_{3}) \cdot \sqrt{3}}{2 \cdot \mathsf{I}_{1} - \mathsf{I}_{2} - \mathsf{I}_{3}}$$
(37)

We have just obtained identical results by calculations via both Phasors and Waves based methods. It is shown above that there is an obvious and solid connection between methods based on waves and phasors, whereby Phasors method is generalized and simplified Waves method.

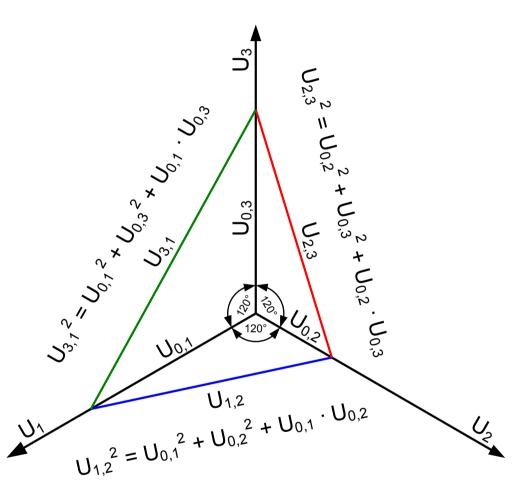
I have personally derived above equation for null current in three-phase system in 1995. My house possesses three-phase cords as most ones in Europe, and my novel electric boiler required calculation of wires' cross-sections including null one in accordance with legal regulations. Therefore, I needed this equation to estimate null-wire cross-section for various malfunctions, including failure of a heater or even two ones when system becomes highly asymmetric making null current nontrivial.

Before that simple and extremely useful equation of mine, there was only one big and unpractical trigonometric equation in the common engineering usage, available only numerically tabulated, usually in a rear and expensive textbooks. I was still student of electrical engineering preparing my diploma thesis in those times and it was quite astonishing to me that such simple application of several trigonometric formulas yielding above equation is so big problem for seemingly prominent experts. Then, after a while, I realized that discoveries of giants in physics remain discovered and what they missed to discover remain undiscovered and therefore any improvement is not advisable because this pales glowing of these alleged giants altogether with the authority of professors relying on them and mainly repeat their teaching without any reassessing at all.

I possess nearly 50 books of energetic, mainly on English, and none of them contains any explanation why three-phase AC grid is commonly used in power transmission, literally none of them does contain abovementioned explanation. It is hard to believe that nobody was willing to analyze and find the true reason for utilization of this terrific power transmission system. Most homes in Europe have three-phase cords, the ordinary residential voltage is 230V between any phase and null and 400V between any two phases ($230 \cdot \sqrt{3} \approx 400$). Voltage between two phases is:

$$U_{1,2} = \sqrt{U_{0,1}^{2} + U_{0,2}^{2} + U_{0,1} \cdot U_{0,2}}$$
(38)

Geometric representation of the merged phases is depicted the following picture: Fig. 4



Above picture depicts situation with three-phases with all mutual angles of 120° creating three distinctive equations:

$$U_{1,2}^{2} = U_{0,1}^{2} + U_{0,2}^{2} + U_{0,1} \cdot U_{0,2}$$
(39)

$$U_{2,3}^{2} = U_{0,2}^{2} + U_{0,3}^{2} + U_{0,2} \cdot U_{0,3}$$
(40)

$$U_{3,1}^{2} = U_{0,1}^{2} + U_{0,3}^{2} + U_{0,1} \cdot U_{0,3}$$
(41)

This system consisted of above three relatively simple equations define voltages between phases and null wire. Oftentimes the null wire is not available and then these voltages between phases and the null can be derived theoretically. Above

three equations form non-linear system that can be explicitly solved to obtain voltage between phases and not available null wire. By careful elimination of variables above system collapses to biquadrate equation that is going to be solved on $U_{0,3}$ and therefore solution on $U_{0,3}^{2}$ is:

$$U_{0,3}^{2} = \mp \frac{\sqrt{3 \cdot \left(\left(U_{1,2}^{2} + U_{2,3}^{2} + U_{3,1}^{2}\right)^{2} - 2 \cdot \left(U_{1,2}^{4} + U_{2,3}^{4} + U_{3,1}^{4}\right)\right) \cdot \left(U_{1,2}^{2} - U_{2,3}^{2}\right)^{2} \cdot \left(U_{1,2}^{2} - U_{3,1}^{2}\right)^{2}}{3 \cdot \left(3 \cdot \left(U_{1,2}^{4} + U_{2,3}^{4} + U_{3,1}^{4}\right) - \left(U_{1,2}^{2} + U_{2,3}^{2} + U_{3,1}^{2}\right)^{2}\right)} +$$

$$(42)$$

$$+\frac{4 \cdot U_{1,2}^{4} \cdot \left(\!U_{2,3}^{2} + U_{3,1}^{2}\right) - U_{1,2}^{2} \cdot \left(5 \cdot \left(\!U_{2,3}^{2} + U_{3,1}^{2}\right)^{\!2} - 7 \cdot U_{2,3}^{2} \cdot U_{3,1}^{2}\right) + U_{2,3}^{2} \cdot U_{3,1}^{2} \cdot \left(\!U_{2,3}^{2} + U_{3,1}^{2}\right) + U_{2,3}^{2} \cdot \left(\!U_{2,3}^{2} + U_{3,1}^{2}\right) + U_{2,3}^{2} \cdot \left(\!U_{2,3}^{2} + U_{2,3}^{2}\right) + U_{2,3}^{2} \cdot \left(\!U_{2,3}^{2} + U_{3,1}^{2}\right) + U_{2,3}^{2} \cdot \left(\!U_{2,3}^{2} + U_{2,3}^{2}\right) + U_{2,3}^{2} +$$

$$+\frac{2\cdot U_{2,3}^{6}+2\cdot U_{3,1}^{6}-U_{1,2}^{6}}{3\cdot \left(3\cdot \left(U_{1,2}^{4}+U_{2,3}^{4}+U_{3,1}^{4}\right)-\left(U_{1,2}^{2}+U_{2,3}^{2}+U_{3,1}^{2}\right)^{2}\right)}$$

The alternating sign of first term should be chosen to maintain all three AC effective voltages positive. With known $U_{0,3}$ unknown $U_{0,1}$ and $U_{0,2}$ are easily calculable by:

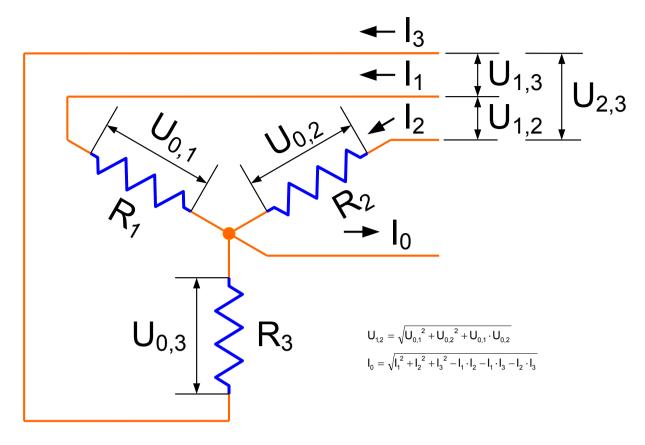
$$U_{0,1} = \frac{\sqrt{4 \cdot U_{3,1}^{2} - 3 \cdot U_{0,3}^{2}} - U_{0,3}}{2}$$
(43)

And:

$$U_{0,2} = \frac{\sqrt{4 \cdot U_{2,3}^{2} - 3 \cdot U_{0,3}^{2}} - U_{0,3}}{2}$$
(44)

Above equation may show the degree of eccentricity of a three-phase system. There is no way to assess potentials between null wire and phases for irregular three-phase system and therefore above equation is valid only for the system without phase asymmetry (all angles are precisely 120°) otherwise it has complex solutions. This equation may be used as prompt test whether the three-phase system without null wire is either regular or irregular one because phase asymmetry can severely damage three-phase asynchronous motors. The equation is dealing with the situation depicted on the following figure:

Fig. 5



Three-phase power line is by definition the line with three wires (three-phases only) or four wires (three-phases and null wire) conducting currents mutually shifted exactly for 120° or $\frac{2}{3}$ · π radian, graphically it is $\frac{1}{3}$ of half-period.

In the case of regular three-phase system with null wire it can be shown that maximal current trough null wire in case of worst asymmetry cannot exceed the current running trough phases and therefore the cross section of all four wires should be identical and it is maximal when current is running trough one phase only. This case usually occurs only by malfunction of a heater or a winding on some phase.

ELECTRIC MOTORS

I have divided all electric motors into two main groups only: motors utilizing magnet-magnet interaction and motors utilizing magnet-current interaction. Magnet-magnet interaction is immensely stronger than magnet-current interaction that is essentially Lorentz's force. DC motors have magnet-magnet interaction while AC motor and DC Faraday's disk have magnet-current interaction. Advanced contemporary electric motors are based on magnet-magnet interaction with construction usually more expensive than one based on magnet-current interaction due to mandatory usage of expensive niobium permanent magnet in rotor, while advance AC motors use only purified copper with improved conductivity.

(3) ASYNCHRONOUS MOTOR

Asynchronous motor is genuine invention of Nikola Tesla and this invention effectively utilizes AC power in mechanical applications. Although it is possible to drive liner brush DC motors (with electromagnets in both rotor and stator) with AC current (as it is common in ordinary usage propelling vacuum cleaners, hair dryers, mixers and other home appliances, and recently in washing machines too replacing bigger and quieter asynchronous ones) they are not that efficient as full-fledged AC motors. The efficiency of brush motors supplied by AC current can be drastically improved by utilization of rectifier bridges bringing DC current back to these motors and by the rectification thermal loss of eddy currents is practically eliminated from stators coils. These coils are supplied now with the DC current that cannot induce eddy currents into the ferromagnetic cores of stator's coils.

Brush DC motors require maintenance, regular brush replacement and they are remarkably noisy due to sparkly brushes too. Contrary to them, three-phase AC motors do not heave wearing parts, therefore they do not require regular maintenance making AC motor extremely durable and yet it has constant torque on the shaft just like DC Faraday wheel's motor and it is almost silent. On small Three-phase AC motors constant torque can be even manually detected. Mono-phase asynchronous motor has one wearing part embodied in the electrolytic capacitor providing additional phase, while it is essentially two-phase motor with additional phase delayed by this capacitor. AC motor is almost silent in respect to sparkly brushes in DC motor where mostly those sparks create intensive noise. As aforementioned, the main advantage of AC motor is completely constant torque like in case of DC Faraday homopolar wheel motor, which extremely elongates lifespan of cogs used in rest of machinery. Disadvantage is small torque on low RPM, as it is shown by the following function:

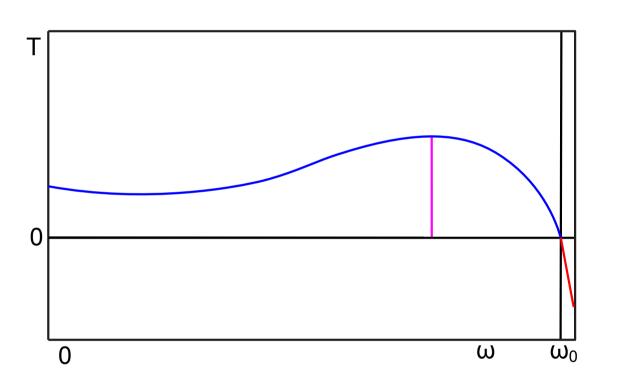


Fig. 6

Asynchronous motors also have smaller power density (kg/W) than brush DC motors, simply because torque is created by magnet-current interaction instead of much stronger magnet-magnet interaction utilized in brush DC motors. AC motors also have two gaps in magnetic circuit per magnet while DC motors have only one gap per magnet. This means that for same power AC motor is much bigger than DC motor and therefore electric automobiles currently mostly utilize much more expensive DC motors instead of much cheaper AC ones.

DC motor without brushes requires utilization of permanent magnets in rotor and prize of such permanent magnet made of expensive alloy or also expensive niobium makes great portion of the prize of entire vehicle, which is much more expensive than one utilizing much heavier AC motor.

For proper understanding of AC motor operation, i.e. modus operandi, we need to expound it on several stages, each clearly conceivable by the appropriate thought experiments.

LORENTZ FORCE

Lorentz force is the basic force propelling all electric motors based on magnet-current interaction. It is essentially represented by the shaft between two magnets with the electric current running trough it, as depicted on the following figure:

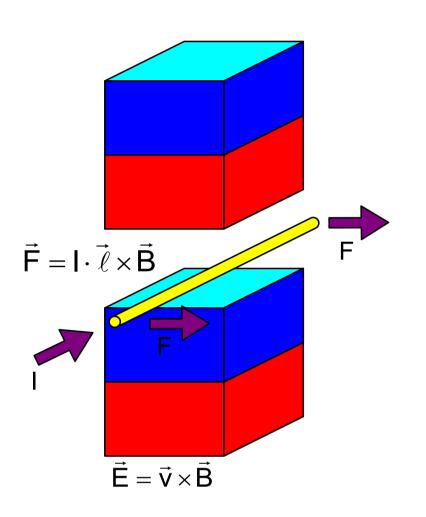
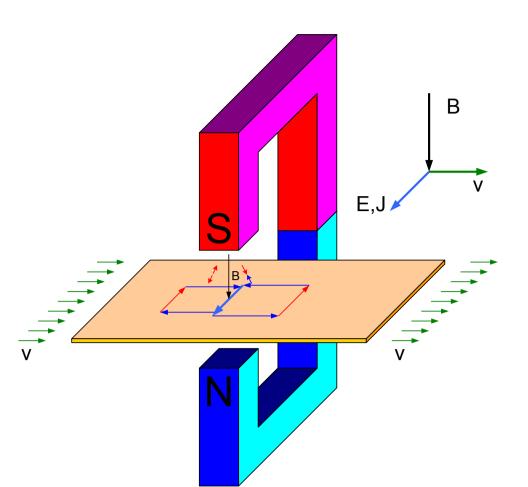


Fig. 7

Lorentz force is quantified with the following formula:

$$d\vec{F}_{1,2} = dQ1 \cdot \vec{v}_{1,2} \times \vec{B}_2 = -I \cdot \vec{B}_1 \times d\vec{\ell}_2$$
(45)

When we drug a magnet over the conductive sheet, the magnet will try to pull the sheet with itself due to electromechanical coupling:



The moving magnet generates the electric current running perpendicularly to its motion, creating two current loops, one in front the magnet and one behind the magnet. This current running beneath and above the C magnet ends creates mechanical interaction between the sheet and the magnet, i.e. force proportional to the differential velocity, more conveniently this is the velocity between the magnet and the sheet:

$$d\vec{F}_{1,2} = \rho \cdot \vec{v}_{1,2} \times \vec{B}_2 \cdot dV_1 = \vec{J}_1 \times \vec{B}_2 \cdot dV_1 = \sigma \cdot \vec{E}_1 \times \vec{B}_2 \cdot dV_1$$
(46)

 \Rightarrow

$$d\vec{F}_{1,2} = \sigma \cdot \vec{E}_1 \times \vec{B}_2 \cdot dV_1 = \sigma \cdot \left(\vec{v}_{1,2} \times \vec{B}_2\right) \times \vec{B}_2 \cdot dV_1$$
(47)

Whereas dF_{12} is force acting to the conductive sheet, v_1 is velocity of the sheet, σ is electric conductivity of the sheet, B_2 is magnitude of the magnetic field pervading the conductive sheet and dV is infinitesimal volume of the sheet. The force between the magnet and sheet can be augmented by improvement of the sheet's conductivity, where better conductivity implies stronger force. Finally we is going to simplify above equation based on fact that B_1 is mostly perpendicular to $v_{1,2}$:

$$d\vec{F}_{1,2} = \sigma \cdot \left(\left(\vec{B}_2 \cdot \vec{v}_1 \right) \cdot \vec{B}_2 - \vec{B}_2^2 \cdot \vec{v}_1 \right) \cdot dV_1 \approx -\sigma \cdot \vec{B}_2^2 \cdot \vec{v}_{1,2} \cdot dV_1$$
(48)

Above equation states that mechanical coupling between the magnet and the sheet is linear, while previous figure depicts non-linear functional connection. This is explained by induction of the counter-magnetic field in the sheet that is confronted to the external one making the whole mechanism of interaction highly nonlinear.

The curve depicted on Fig. 6 can be theoretically derived with assumption that electric current induced into the conductive sheet depicted on Fig. 8 generate magnetic field opposite to the external one diminishing it:

$$\vec{B}_{int} = \vec{B}_{ext} - \vec{B}_{ind}$$
(49)

Fig. 8

Above formula states that internal magnetic field originated in the field of external C permanent magnet is diminished by the amount of induced one. Induced magnetic field is:

$$\Phi = \mathsf{L} \cdot \mathsf{I} \tag{50}$$

 \Rightarrow

$$\vec{B}_{ind} \cdot d\vec{S} = L \cdot dI \tag{51}$$

 \Rightarrow

$$\vec{\mathsf{B}}_{\text{ind}} = \vec{\mathsf{L}} \times \vec{\mathsf{J}} = \vec{\mathsf{L}} \times \left(\boldsymbol{\sigma} \cdot \vec{\mathsf{E}} \right) \tag{52}$$

 \Rightarrow

$$\vec{B}_{ind} = \sigma \cdot \vec{L} \times \left(\vec{v} \times \vec{B}_{int} \right)$$
(53)

Above equation can be converted into scalar form due to fact that vectors values are mutually perpendicular:

$$\mathbf{B}_{ind} = \boldsymbol{\sigma} \cdot \mathbf{L} \cdot \mathbf{B}_{int} \cdot \mathbf{v} \tag{54}$$

According (49) it is:

$$\mathbf{B}_{\text{int}} = \mathbf{B}_{\text{ext}} - \mathbf{B}_{\text{ind}} = \mathbf{B}_{\text{ext}} - \boldsymbol{\sigma} \cdot \mathbf{L} \cdot \mathbf{B}_{\text{int}} \cdot \mathbf{v}$$
(55)

 \Rightarrow

$$B_{int} = \frac{B_{ext}}{1 + \sigma \cdot L \cdot V}$$
(56)

Whereas B_{int} is magnetic field in the sheet, B_{ext} is external magnetic field that pervades the sheet, σ is electric conductance where $\sigma = \rho^{-1}$ and L is inductance of the sheet which may be considered as a constant characteristic for the particular sheet.

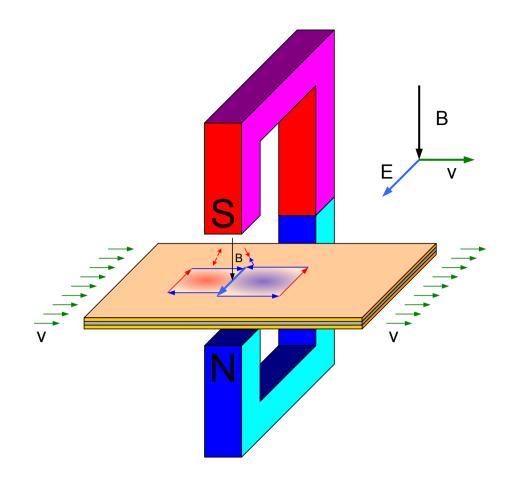
Force acting to the sheet according following equation of mine is:

$$\frac{dF}{dV} = \sigma \cdot B_{int}^{2} \cdot v = \sigma \cdot \left(\frac{B_{ext}}{1 + \sigma \cdot L \cdot v}\right)^{2} \cdot v$$
(57)

Whereas F is drag force created by the external magnetic field acting to the conductive sheet, σ is electric conductance of the sheet, L is inductance of the sheet, v is velocity of the sheet in respect to the source of magnetic field B_{ext} i.e. magnet and V is volume of the sheet pervaded by the external magnetic field B_{ext}. Above formula accurately matches experimental curve depicted on Fig. 6.

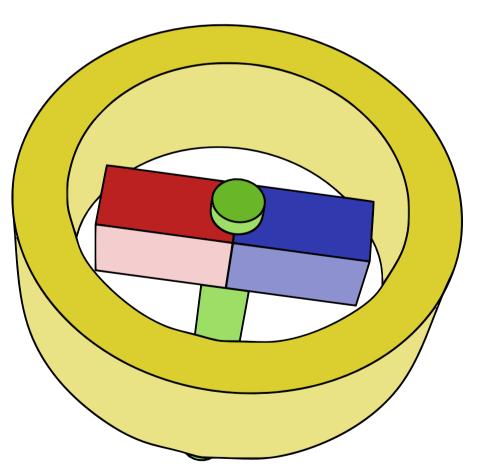
If we split conductive layer into two ones and then put non-conductive ferromagnetic layer between them, then this ferromagnetic layer will create two magnets from two current loops already existing in the conductive layers. One magnet will attract permanent magnet and another one will repulse it, and this effect would significantly increase interaction between the magnet and the encompassed plate:

Fig. 9

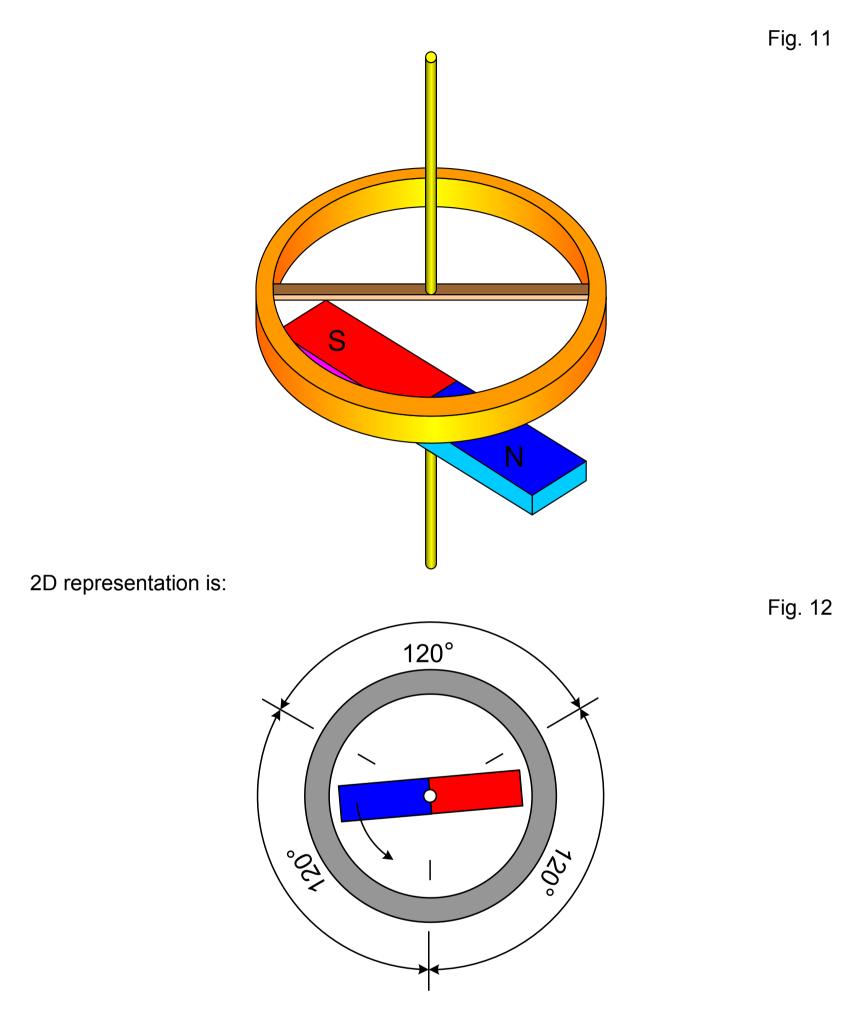


Ferromagnetic materials are usually bed conductors, while good conductors usually have pure ferromagnetic properties, and therefore separating this two roles into two highly specialized layers greatly increases coupling between magnet and the sheet also reducing number of gaps per magnet from 2 to 1.

If we put such magnet inside the conductive ring, then its rotation will create mechanical coupling between the magnet and the rim and this is known as magnetic clutch: Fig. 10

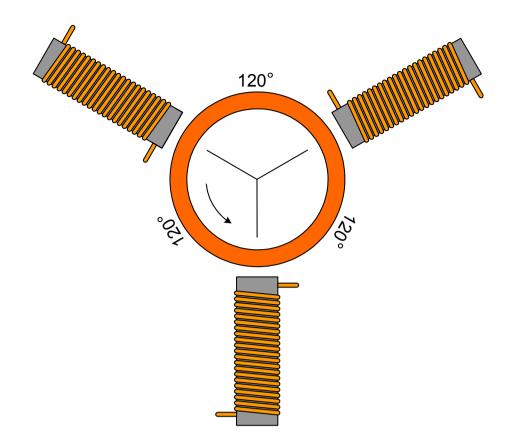


Didactic presentation with magnet below the conductive ring is:



If we replace this single bar magnet with three electromagnets mutually declined for 120°, the resulting magnetic field B will be identical to one of the rotating real permanent bar magnet creating also identical mechanical coupling, i.e. torque between this virtual bar magnet and the real conductive rim, which is essence of the operation of asynchronous motor, i.e. modus operandi:

Fig. 13



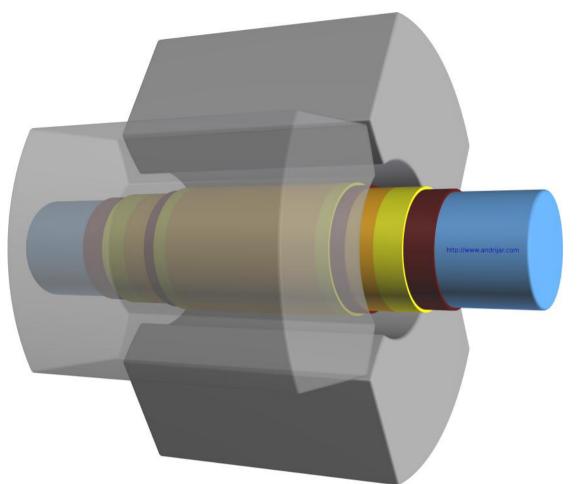
It is important to be noticed that eddy current is necessary ingredient for the functioning of asynchronous motor because this particular current creates rotor's torque. Eddy current is unwonted in the case of DC motor because there eddy current creates only detrimental thermal loss and nothing else. Therefore, the directives for the designing of efficient AC and DC electric motors are completely different ones. This situation was recently changed by the electric car manufacturers that desperately needed to replace expensive DC motors containing rear ferromagnetic niobium with much chipper AC motors, and this required meticulous new design based on all relevant factors instead of just copying and adaptation of DC motor design. Brushed DC motor preceded AC motor for fifty years and when AC motor appeared on market DC motor was already evolved product. The fact is that electric motors were used mainly on railway and naval applications because electricity was producing by inefficient dynamo generators and this situation was exceeded by Siemens invention of self-excited DC generator. This particular invention instantly created electric tramways, electric subway and electric locomotives. It was used by Edison as basis for his DC electric grid. It is interesting to be noticed that electric locomotives predated diesel ones.

MAJOR GUIDELINES IN THE DESIGN OF ELECTRIC MOTORS

As aforementioned, there are two major guidelines in the design of electric motor: the type of the interaction and the number of the air gaps intersecting the magnetic circuit per magnet. There are two major types of interactions in electric motors: magnet-current interaction and magnet-magnet one. Magnet-magnet is much stronger interaction than magnet-current one. In brushed DC motor the major applied interaction is magnet-magnet interaction while in asynchronous motor, printed DC motor and Faraday homopolar wheel motor are utilized magnet-current interaction. Magnet-current interaction is essentially Lorentz force, while magnet-magnet interaction is attraction or repulsion of two magnets and therefore brushed DC electric motors have much higher power density (W/kg) than AC asynchronous motors.

I tried to surpass this problem by separation of the magnetic and conductive layers enabling magnetization of magnetic layer, as it is case in DC motor by the magnetic field generated in the **ferromagnetic** layer induced by current running through adjacent **conductive** layer:

Fig. 14



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Brown layer is non-conductive ferromagnetic, **orange** is conductive layer made of copper and terminated with **yellow** ring of gold or silver, this ring may be omitted. Three electromagnets are **grayish**. Idea is to establish magnet-magnet interaction by polarization of the ferromagnetic layer. Ferromagnetic materials are not good conductors and good conductors are not ferromagnetic ones, therefore ideal situation is to separate this layers into two individual ones, each specialized for its purpose, one for conduction and another one for magnetization. This is exclusive design of mine and it is copyright protected.

Gap in the magnetic circuit is the worst enemy of the power density of any electric motor because the gap has immensely smaller magnetic permittivity than ferromagnetic material in the motor. DC motor has one gap per magnet (because there are two magnets in both rotor and stator), while AC motor has two gaps per magnet disrupting magnetic circuit. Gap must be maintained as narrow as possible: if it is too narrow then thermal dilatation of the rotor can cause scratching the stator and too narrow gap aggravates air cooling of the rotor which may cause loosing magnetic properties of rotor by reaching Curie temperature which is pretty low for most of ferromagnetic materials, around 200°C.

Modus operandi of AC motor is much more complicate than DC one, but in return its design offers much simpler construction without wearing sparkly contacts that inject current into the rotor's electromagnets because this current is wirelessly induced in the rotor of AC motor as eddy currents.

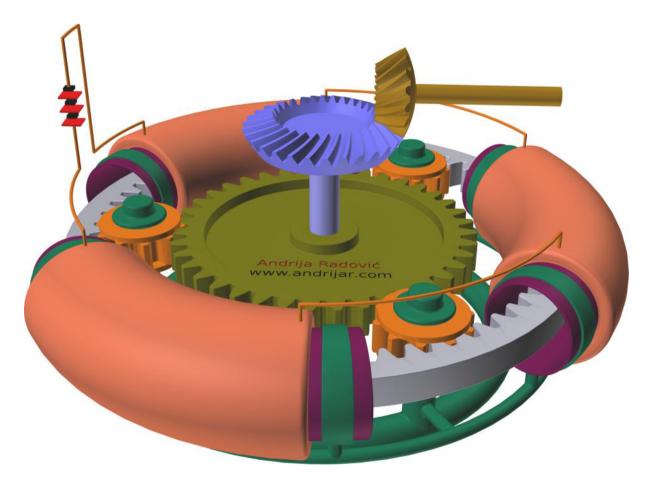
The main objectives in the design of any AC motor is to have good torque on low RPM, i.e. to able to start in various conditions, to be linear, to have big power density (W/kg), to be durable and to be cheap, possibly without rear and expensive materials which is not always attainable combination. We should also bear in mind that good conductors are not ferromagnetic materials and good ferromagnetic materials are bad conductors and sometimes are even insulators. Therefore, polarization of ferromagnetic core in magnet-magnet interaction is done by copper layer and every material is doing its separate role combined. The energy density is very close to DC electric motors.

This was almost all that I could say about the essence of AC motor's operation and the rest of the guild secret is embodied in various technological finesses only, but these finesses make contemporary AC electric motors at least five times more efficient than the early ones, although basic principle of operation essentially remained unchanged during all these decades.

BRUSHLESS DC MOTOR

There is general belief that construction of DC brushless motor is impossible and futile endeavor and it indeed seems to be so according Maxwell equations. The second Maxwell's equation does not possess the term dealing with magnetic current stating that this term should not exist due to absence of magnetic monopoles. I was designed following motor just to show once for all whether N or M hypothesis is the duly correct one and whether this term should retain in those equations regardless there is missing magnetic monopoles or not:

Fig. 15



This motor is consisted of three hollow bended solenoids and one circular permanent magnet with teeth relying on three cogs. The permanent magnet itself is tangentially polarized, identically as it is obtained by joining two bended permanent bar magnets together with matching ends S-N and N-S.

By starting DC current to run in solenoids' windings, those solenoids try to expel this circular permanent magnet out of them, but it is not possible because it is infinite from the viewpoint of those solenoids actually creating propulsion of this permanent magnet. This particular situation is not manageable by classical Maxwell equations' pair due to missing of magnetic current term although it is obvious that circulation of this magnet does create flow of magnetic current. This situation is duly manageable within N hypothesis because total time derivative is able to propagate to the coordinates creating velocity, contrary to partial time one characteristic for M hypothesis that is unable to yield velocity of any kind. It is also obvious that magnetic field of the permanent magnet does move altogether with the magnet interacting with the current loops in surrounding solenoids. Operation of this motor is much more mysterious than it seems to be at the glance simply because the magnetic circuit is closed and allegedly magnetic field does not cross loops in surrounding solenoids at any point and yet there is propulsion and torque. But, if the magnetic field is assumed to be junction of two electric fields with slightly different velocities modified by Doppler's effect than the modus operandi of this motor becomes perfectly clear.

Modus operandi of the motor is described by the following set of equations:

Basic definition of magnetic flux is:

$$d\Phi = \vec{B} \cdot d\vec{S}$$
(58)

Potential energy of the solenoid in infinitesimal form is:

$$dE = L \cdot I \cdot dI = C \cdot U \cdot dU$$
(59)

Energy density of magnetic field in infinitesimal form is:

$$\frac{dE}{dV} = \frac{\vec{B}^2}{2 \cdot \mu}$$
(60)

 \Rightarrow

$$dE = \frac{\vec{B}^2}{2 \cdot \mu} \cdot dV = \frac{\vec{B} \cdot \vec{B}}{2 \cdot \mu} \cdot d\vec{S} \cdot d\vec{\ell} \approx \frac{\left(\vec{B} \cdot d\vec{S}\right) \cdot \vec{B}}{2 \cdot \mu} \cdot d\vec{\ell}$$
(61)

Force between **s**olenoid and permanent bar **m**agnet protracted trough this solenoid is:

$$\vec{F}_{S,N} \approx \frac{dE}{d\vec{\ell}} \approx \frac{\left(\vec{B} \cdot d\vec{S}\right) \cdot \vec{B}}{2 \cdot \mu} \approx \frac{\Phi_{S} \cdot \vec{B}_{M}}{2 \cdot \mu}$$
 (62)

Alternatively, pure magnetic form is:

$$\vec{F}_{S,M} = \oiint_{S} \frac{\vec{B}_{S} \cdot \vec{B}_{M}}{2 \cdot \mu} \cdot d\vec{S}$$
(63)

PRO AND CONTRA OF AC ELECTRIC GRID

Seemingly, AC electric grid has numerous advantages over DC electric grid, but, actually there are many hidden issues related to AC grid that can be maintained properly. One of them is necessity for synchronization of all generators in the electric grid: all generators between China and France rotate the same turn at the same time just to put + on + and - on -, identically by both speed and phase. If this is not the case then the entire electric grid collapses. Although it seems to be huge disadvantage, actually this happened only few times in the entire history of AC electric grid. Therefore, AC power grids require specialized centers for frequency coordination to harmonize all production facilities in the electric network. DC electric grid does not need phase maintenance, only equalizing voltages of all generators is required, otherwise some of them will operate as motors. In addition, AC power lines generate eddy currents in the ground bellow them. Consequently, they heat the ground bellow the lines creating thermal losses, while DC power line does not generate eddy current in the ground below the line and consequent loss in the terrain is null. The solution is simple, but more expensive and based on replacement of the lines protracting trough air with buried coaxial power lines that do not emanate magnetic field.

The biggest disadvantage of AC electric grids at those times was impossibility of electric cars charging: Tesla unwillingly exterminated electric cars by his AC electric grid simultaneously heavily supporting novel oil industry. Cupprox (i.e. copper-oxide) rectifier was invented too late to save electric cars by enabling them to be charged in residential AC electric grid again. Rotary AC-DC rectifiers were too noisy, too big, and inefficient and excessively complicate for ordinary home charging of electric cars from AC grid. It is interesting that there is an efficient mechanical solution: rectifying set of brushes on armature driven with synchronous motor, much better than switching rectifiers based on relays that had limited durability due to constantly sparking contacts. They also spent excessive amount of time to realize that any DC motor without permanent magnets can be driven with AC current as stators and rotors electromagnets are changing their polarities simultaneously. Although it seems strange, we should bear in mind that they needed two years to figure out how to make shutting trough airplanes propeller head although this is just one AND mechanical gate – they even put armor to blades to make them resistive to bullets...

AC electric grid replaced the residential gas lines used for the illumination, mechanical power trough piston gas motors and heating systems. The major component of this gas for illumination was highly toxic carbon-monoxide. If the wind or draft extinguished the flame then there was great chance for the entire dormant family is going to be poisoned to death by this toxic carbon monoxide. Carbon-monoxide was produced in huge quantities by incomplete oxidation of wood or coal in gas factories and the system was pretty much refined. It is interesting to be noticed that carbon monoxide flame casts invisible ultraviolet light just as today's neon tubes and therefore the fluorescent layer was necessary to convert this invisible UV light into ordinary visible white one via fluorescent screens, just as it is case with today's neon tubes and neon bulbs.

EXTENDED PYTHAGOREAN THEOREM

Above text revealed that there is geometric connotation of phase mechanics exposing that Pythagorean Theorem can be expanded with several more formulas for triangles with angles of 30°, 45°, 60°, 90°, 120°, 135° and 150°. The triangle that encompasses angle of 120° is especially important for the geometric connotation of three-phase system.

For triangle with encompassed angle of 30° the formula for the side opposite to this angle is:

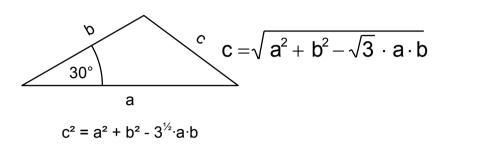
$$c^2 = a^2 + b^2 - \sqrt{3} \cdot a \cdot b \tag{64}$$

Fig. 16

Fig. 17

Fig. 18

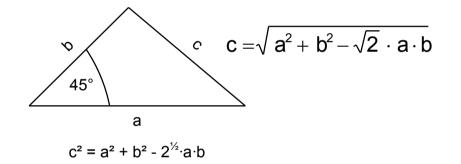
It is depicted on the following image:



For triangle with encompassed angle of 45° the formula of opposite side opposite to this angle is:

$$c^{2} = a^{2} + b^{2} - \sqrt{2} \cdot a \cdot b \tag{65}$$

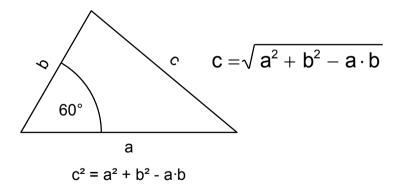
As depicted on the following image:



For triangle with encompassed angle of 60° the formula is quite elegant:

$$\mathbf{c}^2 = \mathbf{a}^2 + \mathbf{b}^2 - \mathbf{a} \cdot \mathbf{b} \tag{66}$$

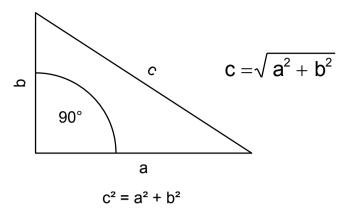
As depicted on the following image:



For triangle with encompassed angle of 90° formula is well known Pythagorean Theorem:

$$c^2 = a^2 + b^2$$
 (67)

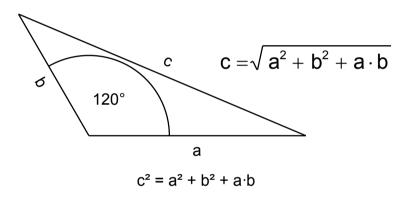
As depicted on the following image:



For triangle with encompassed angle of 120° formula is very important as it deals with geometric representation of regular Three-Phase electric system:

$$\mathbf{c}^2 = \mathbf{a}^2 + \mathbf{b}^2 + \mathbf{a} \cdot \mathbf{b} \tag{68}$$

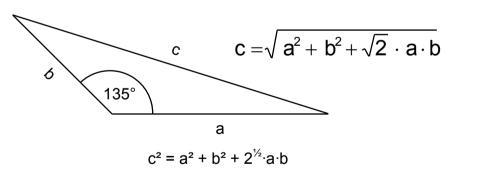
As depicted on the following image:



For triangle with encompassed angle of 135° formula is:

$$\mathbf{c}^2 = \mathbf{a}^2 + \mathbf{b}^2 + \sqrt{2} \cdot \mathbf{a} \cdot \mathbf{b} \tag{69}$$

As depicted on the following image:



For triangle with encompassed angle of 150° formula is:

$$c^2 = a^2 + b^2 + \sqrt{3} \cdot a \cdot b \tag{70}$$

As depicted on the following image:

24/25

Fig. 19

Fig. 20

Fig. 21

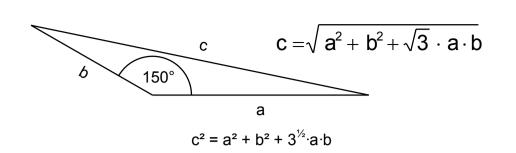


Fig. 22

Above formulas may be very useful for torturing students on exams, as above mentioned, especially one pertaining on the triangle with 120° angle is useful for the case or regular Three-Phase system because it gives to us geometric representation of three-phases' space.

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