

GENERAL VECTORS IDENTITIES

Algebraic vector relations:

$$1. \quad \vec{E} = \begin{bmatrix} E_x & E_y & E_z \end{bmatrix} = E_x(x,y,z) \cdot \hat{i} + E_y(x,y,z) \cdot \hat{j} + E_z(x,y,z) \cdot \hat{k}$$

$$2. \quad \vec{B} = \begin{bmatrix} B_x & B_y & B_z \end{bmatrix} = B_x(x,y,z) \cdot \hat{i} + B_y(x,y,z) \cdot \hat{j} + B_z(x,y,z) \cdot \hat{k}$$

$$3. \quad \vec{E} + \vec{B} = \begin{bmatrix} E_x + B_x & E_y + B_y & E_z + B_z \end{bmatrix}$$

$$4. \quad \vec{E} \cdot \vec{B} = E_x \cdot B_x + E_y \cdot B_y + E_z \cdot B_z$$

$$5. \quad \vec{E} \times \vec{B} = \begin{bmatrix} E_y \cdot B_z - E_z \cdot B_y & E_z \cdot B_x - E_x \cdot B_z & E_x \cdot B_y - E_y \cdot B_x \end{bmatrix}$$

$$6. \quad \vec{E} \cdot \vec{B} = |\vec{E}| \cdot |\vec{B}| \cdot \cos(\angle(\vec{E}, \vec{B}))$$

$$7. \quad |\vec{E} \times \vec{B}| = |\vec{E}| \cdot |\vec{B}| \cdot \sin(\angle(\vec{E}, \vec{B})) \cdot \hat{n} \quad \text{Whereas } \hat{n} \perp \vec{E} \text{ and } \hat{n} \perp \vec{B}.$$

$$8. \quad \vec{E} \times (\vec{B} + \vec{C}) = \vec{E} \times \vec{B} + \vec{E} \times \vec{C}$$

$$9. \quad \vec{E} + \vec{B} = \vec{B} + \vec{E}$$

$$10. \quad \vec{E} \cdot \vec{B} = \vec{B} \cdot \vec{E}$$

$$11. \quad \vec{E} \times \vec{B} = -\vec{B} \times \vec{E}$$

$$12. \quad (\vec{E} + \vec{B}) + \vec{C} = \vec{E} + (\vec{B} + \vec{C})$$

$$13. \quad |\vec{E}| = \sqrt{\vec{E} \cdot \vec{E}}$$

$$14. \quad \hat{E} = \frac{\vec{E}}{|\vec{E}|}$$

$$15. \quad \vec{E} \cdot \vec{E} = |\vec{E}|^2 = \vec{E}^2$$

$$16. \quad \vec{E}^2 \cdot \vec{B}^2 = (\vec{E} \times \vec{B})^2 + (\vec{E} \cdot \vec{B})^2$$

$$17. \quad \vec{E} \cdot (\vec{B} + \vec{C}) = \vec{E} \cdot \vec{B} + \vec{E} \cdot \vec{C}$$

$$18. \quad \vec{U} \cdot (\vec{A} + \vec{B}) = \vec{U} \cdot \vec{A} + \vec{U} \cdot \vec{B}$$

$$19. \quad \vec{E} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{E} \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{E})$$

$$20. \quad \vec{E} \times (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{E} \cdot \vec{C}) - \vec{C} \cdot (\vec{E} \cdot \vec{B})$$

$$21. \quad \vec{E} \times (\vec{B} \times \vec{C}) = \vec{B} \times (\vec{E} \times \vec{C}) - \vec{C} \times (\vec{E} \times \vec{B})$$

$$22. \quad (\vec{E} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) = (\vec{E} \cdot \vec{C}) \cdot (\vec{B} \cdot \vec{D}) - (\vec{E} \cdot \vec{D}) \cdot (\vec{B} \cdot \vec{C})$$

$$23. \quad \vec{E} \times (\vec{B} \times (\vec{C} \times \vec{D})) = (\vec{B} \cdot \vec{D}) \cdot (\vec{E} \times \vec{C}) - (\vec{B} \cdot \vec{C}) \cdot (\vec{E} \times \vec{D})$$

Differential vector relations:

$$24. \quad \vec{\nabla} = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}$$

$$25. \quad \vec{\nabla} U = \overrightarrow{\text{GRAD}} U = \begin{bmatrix} \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} & \frac{\partial U}{\partial z} \end{bmatrix}$$

$$26. \quad \vec{\nabla} \times = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \times$$

$$27. \quad \vec{\nabla} \times \vec{E} = \overrightarrow{\text{CURL}} \vec{E} = \overrightarrow{\text{ROT}} \vec{E} = \begin{bmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} & \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} & \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{bmatrix}$$

$$28. \quad \vec{\nabla} \cdot \vec{E} = \text{DIV } \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$29. \quad \vec{\nabla}(\vec{E}) = \overrightarrow{\text{GRAD}}(\vec{E}) = \vec{\nabla} \begin{bmatrix} E_x & E_y & E_z \end{bmatrix} = \begin{bmatrix} \frac{\partial E_x}{\partial x} & \frac{\partial E_y}{\partial x} & \frac{\partial E_z}{\partial x} \\ \frac{\partial E_x}{\partial y} & \frac{\partial E_y}{\partial y} & \frac{\partial E_z}{\partial y} \\ \frac{\partial E_x}{\partial z} & \frac{\partial E_y}{\partial z} & \frac{\partial E_z}{\partial z} \end{bmatrix}$$

$$30. \quad \nabla([A]) = \text{DIV}([A]) = \nabla \left(\begin{bmatrix} A_{1,1} & A_{2,1} & A_{3,1} \\ A_{1,2} & A_{2,2} & A_{3,2} \\ A_{1,3} & A_{2,3} & A_{3,3} \end{bmatrix} \right) = \begin{bmatrix} \frac{\partial A_{1,1}}{\partial x} + \frac{\partial A_{1,2}}{\partial y} + \frac{\partial A_{1,3}}{\partial z} & \frac{\partial A_{2,1}}{\partial x} + \frac{\partial A_{2,2}}{\partial y} + \frac{\partial A_{2,3}}{\partial z} & \frac{\partial A_{3,1}}{\partial x} + \frac{\partial A_{3,2}}{\partial y} + \frac{\partial A_{3,3}}{\partial z} \end{bmatrix}$$

$$31. \quad (\vec{E} \cdot \vec{\nabla}) \cdot U = \vec{E} \cdot \vec{\nabla} U$$

$$32. \quad \vec{\nabla}(\vec{E} \cdot \vec{B}) = \vec{E} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{E}) + (\vec{E} \cdot \vec{\nabla}) \cdot \vec{B} + (\vec{B} \cdot \vec{\nabla}) \cdot \vec{E}$$

$$33. \quad \vec{\nabla}(f(U)) = \frac{\partial f(U)}{\partial U} \cdot \vec{\nabla} U$$

$$34. \quad \nabla(\vec{E} + \vec{B}) = \nabla \vec{E} + \nabla \vec{B}$$

$$35. \quad \nabla(U \cdot \vec{E}) = U \cdot \nabla \vec{E} + \vec{E} \cdot \vec{\nabla} U$$

$$36. \quad \nabla(\vec{E} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{B})$$

$$37. \quad \vec{\nabla} \times (\vec{E} + \vec{B}) = \vec{\nabla} \times \vec{E} + \vec{\nabla} \times \vec{B}$$

$$38. \quad \vec{\nabla} \times (U \cdot \vec{E}) = (\vec{\nabla} U) \times \vec{E} + U \cdot \vec{\nabla} \times \vec{E} = U \cdot \vec{\nabla} \times \vec{E} - \vec{E} \times \vec{\nabla} U$$

$$39. \quad \vec{\nabla} \times (\vec{E} \times \vec{B}) = \vec{E} \cdot \nabla \vec{B} - \vec{B} \cdot \nabla \vec{E} + (\vec{B} \cdot \vec{\nabla}) \cdot \vec{E} - (\vec{E} \cdot \vec{\nabla}) \cdot \vec{B}$$

$$40. \quad \nabla(\vec{\nabla} \times \vec{E}) = 0$$

$$41. \quad \Delta = \vec{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$42. \quad \Delta U = \nabla(\vec{\nabla} U) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

$$43. \quad \Delta \vec{E} = \vec{\nabla}^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla} \times (\vec{\nabla} \times \vec{E})$$

$$44. \quad \Delta(U + W) = \Delta U + \Delta W$$

$$45. \quad \Delta(U \cdot W) = W \cdot \Delta U + U \cdot \Delta W + 2 \cdot (\vec{\nabla} U) \cdot (\vec{\nabla} W)$$

$$46. \quad \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \Delta \vec{E} = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E}$$

$$47. \quad \Delta(U \cdot \vec{E}) = U \cdot \Delta \vec{E} + \vec{E} \cdot \Delta U + 2 \cdot ((\vec{\nabla} U) \cdot \vec{\nabla}) \cdot \vec{E}$$

$$48. \quad \vec{\nabla} \times (\vec{\nabla} U) = \vec{0}$$

$$49. \quad \vec{\nabla} \times (\vec{\nabla} \times (U \cdot \vec{E})) = (\vec{\nabla} U) \times (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot \Delta U + (\vec{E} \cdot \vec{\nabla}) \cdot U + U \cdot \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) + (\vec{\nabla} U) \cdot (\vec{\nabla} \times \vec{E}) - ((\vec{\nabla} U) \cdot \vec{\nabla}) \cdot \vec{E}$$

Integral vector relations:

50. $\iiint_V \vec{\nabla} U \cdot d\vec{V} = \oiint_S U \cdot d\vec{S}$ U denotes a spatial scalar function, V is volume and S is surface.

51. $\iiint_V \vec{\nabla} \vec{E} \cdot d\vec{V} = \oiint_S \vec{E} \cdot d\vec{S}$ Gauss¹-Ostrogradsky² theorem, 3D continuum theorem.

52. $\iiint_V (\vec{\nabla} \times \vec{E}) \cdot d\vec{V} = -\oiint_S \vec{E} \times d\vec{S}$ Stokes³ theorem, whereas $\vec{A}$ denotes spatial vector function.

53. $\iint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = \oint_{\ell} \vec{E} \cdot d\vec{\ell}$ Green⁴ theorem.

54. $\iint_S \vec{\nabla} U \times d\vec{S} = -\oint_{\ell} U \cdot d\vec{\ell}$

55. $\oiint_S (\vec{E} \times (\vec{\nabla} \times \vec{B})) \cdot d\vec{S} = \iiint_V ((\vec{\nabla} \times \vec{E}) \cdot (\vec{\nabla} \times \vec{B}) - \vec{E} \cdot (\vec{\nabla} \times (\vec{\nabla} \times \vec{B}))) \cdot dV$

56. $\oiint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = 0$

57. $\iiint_V ((\vec{\nabla} \times \vec{E}) \cdot (\vec{\nabla} \times \vec{B}) - \vec{E} \cdot (\vec{\nabla} \times (\vec{\nabla} \times \vec{B}))) \cdot dV = \oiint_S \vec{E} \times (\vec{\nabla} \times \vec{B}) \cdot d\vec{S}$

¹ Johann Carl Friedrich Gauss (Gauß), 1777 – 1855

² Mikhail Ostrogradsky, 1801 – 1862

³ George Gabriel Stokes, 1819 – 1903

⁴ George Green, 1793 – 1841

$$58. \quad \iiint_V \left(\vec{E} \cdot \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) - \vec{B} \cdot (\vec{\nabla} \times (\vec{\nabla} \times \vec{E})) \right) \cdot dV = \oiint_S \left(\vec{B} \times (\vec{\nabla} \times \vec{E}) - \vec{E} \times (\vec{\nabla} \times \vec{B}) \right) \cdot d\vec{S}$$

$$59. \quad \iiint_V (\nabla \vec{E} \cdot \nabla \vec{B} - \vec{E} \cdot \vec{\nabla} (\nabla \vec{B})) \cdot dV = \oiint_S (\vec{E} \cdot \nabla \vec{B}) \cdot d\vec{S}$$

$$60. \quad \iiint_V (\vec{E} \cdot \vec{\nabla} (\nabla \vec{B}) - \vec{B} \cdot \vec{\nabla} (\nabla \vec{E})) \cdot dV = \oiint_S (\vec{E} \cdot \nabla \vec{B} - \vec{B} \cdot \nabla \vec{E}) \cdot d\vec{S}$$

$$61. \quad \iiint_V (\vec{U} \cdot \nabla \vec{E}) \cdot dV = \oiint_S (\vec{E} \cdot \nabla \vec{B} - \vec{B} \cdot \nabla \vec{E}) \cdot d\vec{S}$$

$$62. \quad \iiint_V (\vec{U} \cdot \nabla \vec{E} + (\vec{E} \cdot \vec{\nabla}) \vec{U}) \cdot dV = \oiint_S \vec{U} \cdot \vec{E} \cdot d\vec{S}$$

$$63. \quad \iiint_V (\vec{B} \cdot \nabla \vec{E} + (\vec{E} \cdot \vec{\nabla}) \vec{B}) \cdot dV = \oiint_S \vec{B} \cdot (\vec{E} \cdot d\vec{S})$$

$$64. \quad \iiint_V (\vec{B} \cdot \nabla \vec{E} + (\vec{E} \cdot \vec{\nabla}) \vec{B}) \cdot dV = \oiint_S \vec{B} \cdot (\vec{E} \cdot d\vec{S})$$

$$65. \quad \iiint_V \vec{\nabla} \vec{U} \cdot \vec{\nabla} \times \vec{E} \cdot dV = \oiint_S \vec{U} \cdot \vec{\nabla} \times \vec{E} \cdot d\vec{S}$$

$$66. \quad \oiint_S \vec{\nabla} \vec{U} \times d\vec{S} = - \oint_{\ell} \vec{U} \cdot d\vec{\ell}$$

$$67. \quad \oiint_S (\vec{\nabla} \vec{U} \times \vec{\nabla} f) \cdot d\vec{S} = - \oint_{\ell} \vec{U} \cdot \vec{\nabla} f \cdot d\vec{\ell}$$

Vector relations between space and time:

$$68. \quad \vec{\omega}_r = \frac{\vec{r} \times \vec{v}}{r^2} = \frac{\vec{r} \times \frac{d\vec{r}}{dt}}{r^2}$$

Angular velocity of the radius vector.

$$69. \quad \vec{\omega}_v = \frac{\vec{v} \times \vec{a}}{v^2} = \frac{\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}}{\left(\frac{d\vec{r}}{dt}\right)^2}$$

Angular velocity of the velocity vector and this is usual mechanical meaning of angular velocity.

$$70. \quad \vec{\omega}_v = \vec{\nabla} \times \vec{v}$$

Angular velocity of the velocity vector rotation in persistent space, this is valid in persistent space only.

$$71. \quad \vec{\omega}_r = \frac{\vec{r} \times \vec{v}}{r^2} = \frac{\vec{r} \times \dot{\vec{r}}}{r^2}$$

Connection between velocity, angular velocity and radius of the curve.

72.
$$\vec{k} = \frac{\vec{v} \times \vec{a}}{|\vec{v}|^3} = \frac{\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}}{\left|\frac{d\vec{r}}{dt}\right|^3}$$

Inverse radius of the curve which matches vector of rotation.

73.
$$\vec{R} = \frac{\vec{v}^2 \cdot \left(\vec{v}^2 \cdot \vec{a} - (\vec{v} \cdot \vec{a}) \cdot \vec{v} \right)}{\vec{a}^2 \cdot \vec{v}^2 - (\vec{a} \cdot \vec{v})^2} = \frac{\left(\frac{d\vec{r}}{dt} \right)^2 \cdot \left(\left(\frac{d\vec{r}}{dt} \right)^2 \cdot \frac{d^2\vec{r}}{dt^2} - \left(\frac{d\vec{r}}{dt} \cdot \frac{d^2\vec{r}}{dt^2} \right) \cdot \frac{d\vec{r}}{dt} \right)}{\left(\frac{d^2\vec{r}}{dt^2} \right)^2 \cdot \left(\frac{d\vec{r}}{dt} \right)^2 - \left(\frac{d^2\vec{r}}{dt^2} \cdot \frac{d\vec{r}}{dt} \right)^2}$$

74.
$$\frac{dn}{dt} = -\vec{\nabla}(\vec{n} \cdot \vec{v})$$

3D continuum equation, whereas $\vec{v}$ is speed of field's displacement and n is spatial density in persistent space.

75.
$$\frac{d\vec{n}}{dt} = -\vec{\nabla} \times (\vec{n} \times \vec{v})$$

2D continuum equation, whereas $\vec{v}$ is speed of field's displacement, $\vec{n}$ is area density of lines that pervades generalized surface in persistent space.

$$76. \quad \frac{d|\vec{E}|}{dt} = \frac{d\vec{E}}{dt} \cdot \hat{E} = \dot{\vec{E}} \cdot \hat{E}$$

Time derivation of magnitude of arbitrary vector $\vec{E}$.

$$77. \quad \frac{d\hat{E}}{dt} = \frac{\hat{E} \times \left(\frac{d\vec{E}}{dt} \times \hat{E} \right)}{|\vec{E}|} = \vec{\omega}_{\vec{E}} \times \hat{E}$$

Time derivation of ort vector of an arbitrary vector $\vec{E}$.

DESCARTES COORDINATE SYSTEM

$$78. \quad \vec{\nabla}U = \overrightarrow{\text{GRAD}}(U) = \frac{\partial U}{\partial x} \cdot \hat{x} + \frac{\partial U}{\partial y} \cdot \hat{y} + \frac{\partial U}{\partial z} \cdot \hat{z}$$

$$79. \quad \nabla \vec{E} = \text{DIV}(\vec{E}) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial}{\partial x} (\vec{E} \cdot \hat{x}) + \frac{\partial}{\partial y} (\vec{E} \cdot \hat{y}) + \frac{\partial}{\partial z} (\vec{E} \cdot \hat{z})$$

$$80. \quad \vec{\nabla} \times \vec{E} = \overrightarrow{\text{CURL}}(\vec{E}) = \overrightarrow{\text{ROT}}(\vec{E}) = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \cdot \hat{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \cdot \hat{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \cdot \hat{z}$$

$$81. \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$82. \quad \Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

CYLINDRICAL COORDINATE SYSTEM:

$$83. \quad \vec{\nabla}U = \overrightarrow{\text{GRAD}}(U) = \frac{\partial U}{\partial r} \cdot \hat{r} + \frac{1}{r} \cdot \frac{\partial U}{\partial \varphi} \cdot \hat{\varphi} + \frac{\partial U}{\partial z} \cdot \hat{z}$$

$$84. \quad \vec{\nabla} \cdot \vec{E} = \text{DIV}(\vec{E}) = \frac{E_r}{r} + \frac{\partial E_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z} = \frac{1}{r} \cdot \frac{\partial}{\partial r}(r \cdot E_r) + \frac{1}{r} \cdot \frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z}$$

$$85. \quad \vec{\nabla} \times \vec{E} = \overrightarrow{\text{CURL}}(\vec{E}) = \overrightarrow{\text{ROT}}(\vec{E}) = \left(\frac{1}{r} \cdot \frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} \right) \cdot \hat{r} + \left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) \cdot \hat{\varphi} + \left(\frac{1}{r} \cdot \left(E_\varphi - \frac{\partial E_r}{\partial \varphi} \right) + \frac{\partial E_\varphi}{\partial r} \right) \cdot \hat{z}$$

$$86. \quad \Delta U = \frac{1}{r} \cdot \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2} \cdot \frac{\partial^2 U}{\partial \varphi^2} + \frac{\partial^2 U}{\partial z^2} = \frac{1}{r} \cdot \frac{\partial}{\partial r} \left(r \cdot \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \cdot \frac{\partial^2 U}{\partial \varphi^2} + \frac{\partial^2 U}{\partial z^2}$$

SPHERICAL COORDINATE SYSTEM:

$$87. \quad \vec{\nabla} U = \frac{\partial U}{\partial r} \cdot \hat{r} + \frac{1}{r} \cdot \frac{\partial U}{\partial \theta} \cdot \hat{\theta} + \frac{1}{r \cdot \text{SIN}(\theta)} \cdot \frac{\partial U}{\partial \varphi} \cdot \hat{\phi}$$

$$88. \quad \vec{\nabla} \vec{E} = \frac{\partial \vec{E}_r}{\partial r} + \frac{1}{r} \cdot \left(2 \cdot \vec{E}_r + \frac{\partial \vec{E}_\theta}{\partial \theta} + \vec{E}_\theta \cdot \text{CTG}(\theta) + \frac{1}{\text{SIN}(\theta)} \cdot \frac{\partial \vec{E}_\varphi}{\partial \varphi} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot \vec{E}_r) + \frac{1}{r \cdot \text{SIN}(\theta)} \cdot \frac{\partial}{\partial \theta} (\vec{E}_\theta \cdot \text{SIN}(\theta)) + \frac{1}{r \cdot \text{SIN}(\theta)} \cdot \frac{\partial \vec{E}_\varphi}{\partial \varphi}$$

$$89. \quad \vec{\nabla} \times \vec{E} = \frac{1}{r \cdot \text{SIN}(\theta)} \cdot \left(\frac{\partial}{\partial \theta} (\vec{E}_\varphi \cdot \text{SIN}(\theta)) - \frac{\partial \vec{E}_\theta}{\partial \varphi} \right) \cdot \hat{r} + \left(\frac{1}{r \cdot \text{SIN}(\theta)} \cdot \frac{\partial \vec{E}_r}{\partial \varphi} - \frac{1}{r} \cdot \frac{\partial}{\partial r} (\vec{E}_\varphi \cdot r) \right) \cdot \hat{\theta} + \left(\frac{1}{r} \cdot \frac{\partial}{\partial r} (\vec{E}_\theta \cdot r) - \frac{1}{r} \cdot \frac{\partial \vec{E}_r}{\partial \theta} \right) \cdot \hat{\phi}$$

$$90. \quad \Delta U = \frac{1}{r^2} \cdot \frac{\partial}{\partial r} \left(r^2 \cdot \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \cdot \text{SIN}(\theta)} \cdot \frac{\partial}{\partial \theta} \left(\text{SIN}(\theta) \cdot \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \cdot \text{SIN}(\theta)^2} \cdot \frac{\partial^2 U}{\partial \varphi^2}$$

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