SAGNAC EFFECT VERSUS MICHELSON-MORLEY EXPERIMENT

Abstract

It seems that Sagnac effect apparently disputes Michelson-Morley Experiment, but its profound analysis shows this is not the case at all. Sagnac effect drives optical gyroscopes while Michelson-Morley experiment is very foundation of the Einstein's Theory of Relativity. It will be shown in the text below that Sagnac effect does not subvert Michelson-Morley experiment on any way. It seems that there is also missed to be noticed that in Sagnac effect's frequency varies too, not only the phase as it is recognized by official derivation of basic Sagnac formula. This frequency variation is the key for concord between Sagnac effect and Michelson-Morley experiment.

ESSENCE OF SAGNAC EFFECT

Sagnac effect is embodied in the variations of both frequency and phase of the light beam subsequently reflected from the rotating set of circularly disposed mirrors, while the Michelson-Morley Experiment firmly claims that such phase variation should not exist at all due to absence of eater.

OFFICIAL DERIVATION

Official formula of Sagnac effect that only varies phase is derived on the following way:

$$\delta t = \frac{\ell}{c-v} - \frac{\ell}{c+v} = N \cdot \frac{2 \cdot \pi \cdot R}{c - \omega \cdot R} - N \cdot \frac{2 \cdot \pi \cdot R}{c + \omega \cdot R} \tag{1}$$

⇒

$$\delta t = 2 \cdot \pi \cdot R \cdot N \cdot \left(\frac{1}{c - \omega \cdot R} - \frac{1}{c + \omega \cdot R} \right) \tag{2}$$

⇒

$$\delta t = \frac{4 \cdot \pi \cdot N \cdot R^2 \cdot \omega}{c^2 - \omega^2 \cdot R^2} \tag{3}$$

⇒

$$\delta\varphi = 2 \cdot \pi \cdot \frac{\delta t}{T_0} = 2 \cdot \pi \cdot f_0 \cdot \delta t = \frac{8 \cdot \pi^2 \cdot f_0 \cdot N \cdot R^2 \cdot \omega}{c^2 - \omega^2 \cdot R^2} \approx \frac{8 \cdot \pi^2 \cdot R^2 \cdot f_0 \cdot N \cdot \omega}{c^2} \tag{4}$$

⇒

$$\delta\varphi \approx 2 \cdot \left(\frac{2 \cdot \pi}{c} \right)^2 \cdot R^2 \cdot f_0 \cdot N \cdot \omega \approx \frac{8 \cdot \pi}{c} \cdot \frac{\left(R^2 \cdot \pi\right) \cdot N \cdot \omega}{\lambda_0} \approx \frac{8 \cdot \pi}{c} \cdot \frac{S \cdot N \cdot \omega}{\lambda_0} \tag{5}$$

Whereas c is speed of light, π is Pythagorean constant, R is radius of the circular optical winding, S is cross section surface area of the winding, f_0 is frequency of the laser, λ_0 is wavelength of the laser, N is number of fibber-optic turns, $\delta\varphi$ is phase shift caused by Sagnac effect and finally ω is angular velocity of this apparatus that should be measured.

This alleged controversy is embodied in the fact that official derivation directly subverts Michelson-Morley experiment and consequently disobeys to Relativistic Velocity Addition Formula. Very first equation (1) in official derivation directly confronts Theory of Relativity by its relaying to classical equation for velocity addition. This misunderstanding occurred by overseeing that Michelson-Morley setup is duly capable to detect acceleration, but cannot detect velocity, and in the case of angular velocity is an acceleration that corresponds to this angular velocity. Therefore, proper Michelson-Morley setup is able to measure angular velocity through measuring variation in light frequency alteration by acceleration.

In the Michelson-Morley experiment the light is bouncing from the mirrors set backward and forward in respect to the direction of motion, while in the case of Sagnac effect all mirrors reflect light beam on the same way, i.e. either forward or backward exclusively, which is possible only in case of rotational motion. Official explanation actually does include assumption of Galilean addition of relativistic velocities embodied in the very first equation of official derivation, directly opposing the Michelson-Morley experiment and consequently the Theory of Relativity itself. This official derivation simply cannot handle Sagnac effect correctly.

ADVANCED DERIVATION

The problem is not related to the Sagnac effect itself, but rather within its official derivation. We can replace optical winding with few mirrors and then with infinite number of ones, therefore we will transform them into one continuous curved mirror, or circumstantially into fiber optic winding. Obvious fact is that the light reflected by these mirrors is modified by Doppler's effect. For proper analysis of the phenomenon it should be divided into several stages and the first one is bouncing of light beam from the perpendicular motional mirror. Let us imagine one light source at rest and one motional mirror traversing the space with velocity v. Let us attach the observer to the mirror and then the observer will notice that the frequency of light beam is changed in respect to Doppler's effect:

$$f_1 = f_0 \cdot \left(1 - \frac{v_{1,0}}{c}\right) \tag{6}$$

The second observer is attached to the light source and they will see:

$$f_2 = f_1 \cdot \left(1 - \frac{v}{c}\right) = f_0 \cdot \left(1 - \frac{v}{c}\right)^2 = f_0 \cdot \left(1 - \frac{2 \cdot v}{c} + \frac{v^2}{c^2}\right) \tag{7}$$

If the beam is reflected on angle φ then we have:

$$f_2 = f_0 \cdot \left(1 - \frac{2 \cdot v \cdot SIN(\varphi)}{c} + \frac{v^2 \cdot SIN(\varphi)^2}{c^2}\right) \tag{8}$$

We may put infinite number of mirrors around the loop and then angle φ is going to be:

$$\delta f = f_2 - f_0 = f_0 \cdot \frac{2 \cdot v \cdot \delta\varphi}{c} \tag{9}$$

⇒

$$\delta f = f_0 \cdot \frac{2 \cdot \omega_{gyro} \cdot r \cdot \delta\varphi}{c} \tag{10}$$

⇒

$$\delta f = f_0 \cdot \frac{2 \cdot \omega_{gyro} \cdot r \cdot \frac{\delta\ell}{r}}{c} \tag{11}$$

⇒

$$\delta f = f_0 \cdot \frac{2 \cdot \omega_{gyro} \cdot \delta\ell}{c} \tag{12}$$

⇒

$$\delta f = 2 \cdot \frac{f_0 \cdot \omega_{gyro} \cdot \ell}{c} \tag{13}$$

⇒

$$\omega_{gyro} = \frac{c}{2 \cdot \ell} \cdot \frac{\delta f}{f_0} = \frac{c}{2 \cdot \ell} \cdot \left(\frac{f_1}{f_0} - 1\right) = \frac{c}{2 \cdot \ell} \cdot \left(\frac{\delta f}{f_0}\right) \tag{14}$$

Alteration of frequency by Doppler's effect is proportional to the length of the optical winding ℓ, angular velocity of the winding ω_{gyro} and initial frequency f_0.

There is no real discrepancy between Michelson-Morley experiment and Sagnac effect simply because in the first case the light is bouncing backward and forward, while in the case of Sagnac gyroscope the light is reflected from the mirrors always in same direction, causing accumulation of the alterations instead of their mutual annulment.

Accumulation of the phase shift is caused by slight frequency alteration done by Doppler's effect:

$$f_0 = \frac{\omega_0}{2 \cdot \pi} \tag{15}$$

And:

$$SIN(\omega_0 \cdot t + \delta\omega \cdot t) = SIN(\omega_0 \cdot t + \delta\varphi) \tag{16}$$

⇒

$$\delta\varphi = \delta\omega \cdot t = 2 \cdot \pi \cdot \delta f \cdot t = 2 \cdot \pi \cdot \delta f \cdot \frac{\ell}{c} = \frac{2 \cdot \pi \cdot \ell}{c} \cdot \delta f = \frac{4 \cdot \pi \cdot \ell^2 \cdot f_0 \cdot \omega_{gyro}}{c^2} \tag{17}$$

⇒

$$\delta\varphi = \frac{4 \cdot \pi \cdot (2 \cdot \pi \cdot R \cdot N)^2 \cdot f_0 \cdot \omega_{gyro}}{c^2} \tag{18}$$

$\Rightarrow$

$$\delta\varphi = \frac{16 \cdot \pi^2 \cdot \left(\pi \cdot R^2\right) \cdot N^2 \cdot f_0 \cdot \omega_{gyro}}{c^2} = \frac{16 \cdot \pi^2 \cdot S \cdot N^2 \cdot f_0 \cdot \omega_{gyro}}{c^2} \tag{19}$$

$\Rightarrow$

$$\omega_{gyro} = \frac{c^2}{16 \cdot \pi^2 \cdot S \cdot N^2 \cdot f_0} \cdot \delta\varphi \tag{20}$$

Above equation states that phase shift is rather caused by the slight variation of reflected light beam frequency than by the path extension by rotation, which is also duly impossible according Lorentz transformations, Michelson-Morley experiment and Theory of Relativity combined. Therefore there is no essential discrepancy between Michelson-Morley experiment and Sagnac effect at all, there was only misunderstanding of the effect's essence.

DERIVATION BY RELATIVISTIC DOPPLER'S FORMULA

Relativistic Doppler effect is quantified by the following formula:

$$f_1 = f_0 \cdot \frac{\sqrt{1 - \frac{{v_{1,0}}^2}{c^2}}}{1 + \frac{v_{1,0}}{c} \cdot \cos(\varphi)} = f_0 \cdot \frac{\sqrt{1-\beta^2}}{1 + \beta \cdot \cos(\varphi)} \tag{21}$$

Above formula covers both longitudinal and transversal cases of relativistic Doppler effect and this situation is when angle φ tends to be zero:

$$f_1 = f_0 \cdot \sqrt{\frac{1 - \frac{v_{1,0}}{c}}{1 + \frac{v_{1,0}}{c}}} \approx \left(1 - \frac{v}{c}\right) \cdot \left(1 + \frac{v^2}{2 \cdot c^2} + \frac{3 \cdot v^4}{8 \cdot c^4} + \frac{5 \cdot v^6}{16 \cdot c^6} + \frac{35 \cdot v^8}{128 \cdot c^8}\right) \tag{22}$$

Taylor approximation yields identical result as in the case of classical derivation of optical Doppler's effect as the first terms is identical to one of classical derivation.

CONCLUSION

Proper derivation of Sagnac effect and its profound explanation show that it is indeed in perfect agreement with the Michelson-Morley experiment and that there is no discrepancy of any sort at all.

SAGNAC EFFECT AND EINSTEIN THEORY OF RELATIVITY

Equation (20) quantifies Sagnac effect in its optical gyroscope application, i.e. as device for measuring of angular velocity. Therefore it still remains unclear in respect to what Sagnac angular velocity is measured on, which is the issue affecting Classical Mechanics centrifugal force too:

$$F = m \cdot \omega^2 \cdot R \tag{23}$$

⇒

$$\omega = \sqrt{\frac{F}{m \cdot R}} \tag{24}$$

It is perceivable that on ground level the Earth itself should be taken as reference for this angular velocity, but in deep space there is no clear referential point for measurement of this rotation. Actually, celestial mechanists use center of mass of planetary system as referential point for determination of the acceleration in Second Newtonian law without any real reason for that except the lack of anything else suitable. Recent official explanation is that ubiquitous Higgs field pervading everything gives inertia to all bodies in universe and that this interaction is quantized by Higgs boson, which all combined confronts Theory of Relativity claiming that inertia and gravitation are the same phenomena and obviously they are not. My humble explanation is that initially there were two black holes, one filled with matter and another one filled with antimatter, and then one filled with matter exploded in event known as Big Bang while other one remained intact retaining its own gravitational field that gives inertia to all bodies in universe and also pushing all matter away from it causing well known Hubble expansion. I have derived the formula that explains that essence of inertia is originated in external field that pervades the masses or charges:

$$\vec{F}_{inertial} = \frac{V_{external} \cdot \vec{a}}{c^2} \cdot Q = \frac{V_{external} \cdot \vec{a}}{c^2} \cdot m \tag{25}$$

In case of mass this external field is much stronger of fields of adjacent bodies than in case of electromagnetism where we are dealing with effective masses of charges indeed.

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